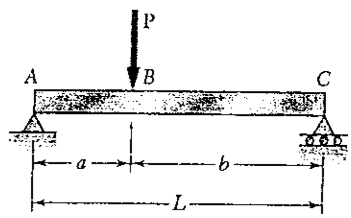
Chapter 5

Problem 5.1

5.1 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

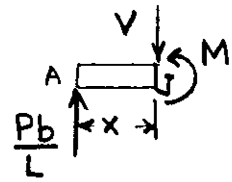


Reactions

$\Sigma M_C = 0 \qquad LA - bP = 0 \qquad A = \frac{Pb}{L}$

$\Sigma M_A = 0 \qquad LC - aP = 0 \qquad C = \frac{Pa}{L}$

From A to B $\qquad 0 < x < a$

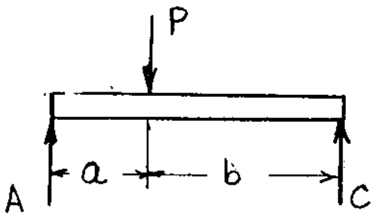


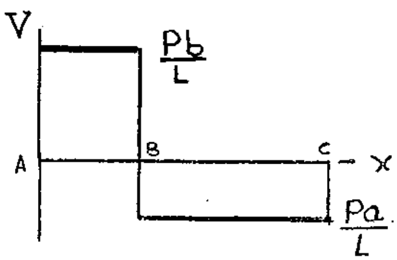
$+\uparrow \Sigma F_y = 0 \qquad \frac{Pb}{L} - V = 0$

$V = \frac{Pb}{L}$

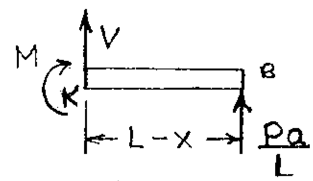
$\circlearrowleft \Sigma M_J = 0 \qquad M - \frac{Pb}{L}x = 0$

$M = \frac{Pbx}{L}$





From B to C $\qquad a < x < L$

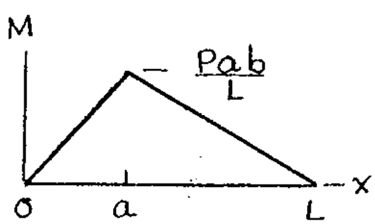


$+\uparrow \Sigma F_y = 0 \qquad V + \frac{Pa}{L} = 0$

$V = -\frac{Pa}{L}$

$\circlearrowleft \Sigma M_K = 0 \qquad -M + \frac{Pa}{L}(L - x) = 0$

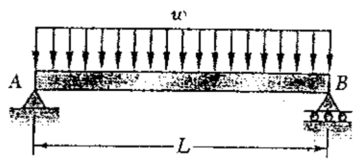
$M = \frac{Pa(L-x)}{L}$



At section B $\qquad M = \frac{Pab}{L}$

Problem 5.2

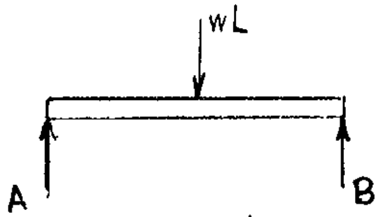
5.2 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.



Reactions

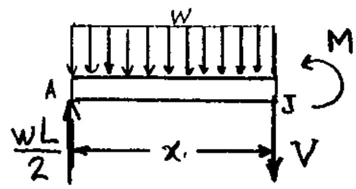
$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$

$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$

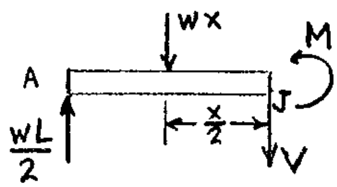


Free body diagram for determining reactions

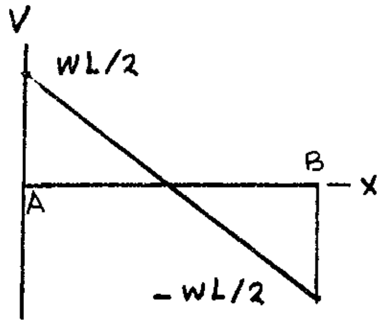
Over whole beam $\quad 0 < x < L$



Place section at x.



Replace distributed load by equivalent concentrated load.



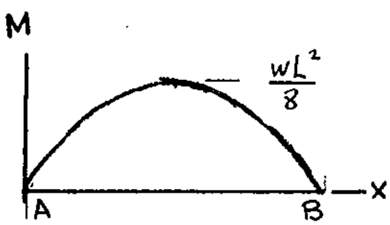
$+\uparrow \Sigma F_y = 0 \qquad \frac{wL}{2} - wx - V = 0$

$$V = w\left(\frac{L}{2} - x\right)$$

$\circlearrowleft \Sigma M_J = 0 \qquad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$

$$M = \frac{w}{2}(Lx - x^2)$$

$$= \frac{w}{2}x(L - x)$$

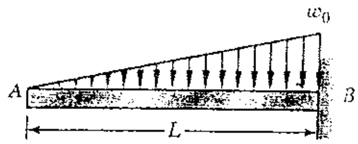


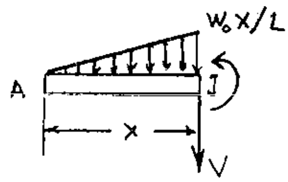
Maximum bending moment occurs at $x = \frac{L}{2}$.

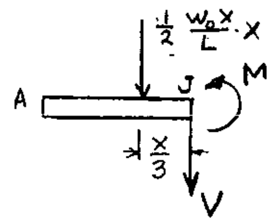
$$M_{max} = \frac{wL^2}{8}$$

Problem 5.3

5.3 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

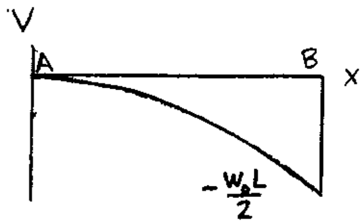






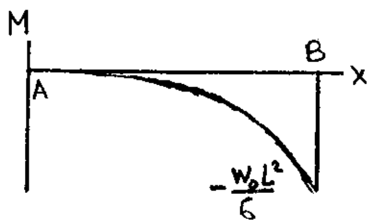
$+\uparrow \Sigma F_y = 0 \qquad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

$$V = -\frac{w_0 x^2}{2L}$$



$\circlearrowleft \Sigma M_J = 0 \qquad \frac{1}{2}\frac{w_0 x}{L}\cdot x \cdot \frac{x}{3} + M = 0$

$$M = -\frac{w_0 x^3}{6L}$$

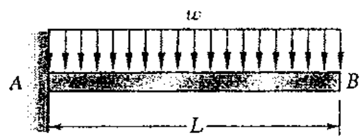


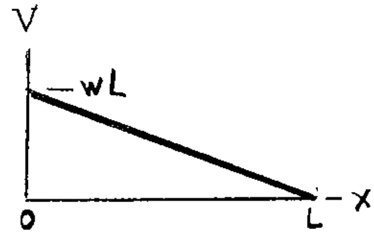
At $x = L \qquad V = -\frac{w_0 L}{2} \qquad |V|_{max} = \frac{w_0 L}{2}$

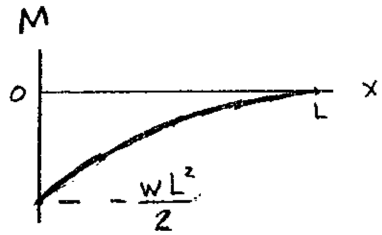
$M = -\frac{w_0 L^2}{6} \qquad |M|_{max} = \frac{w_0 L^2}{6}$

Problem 5.4

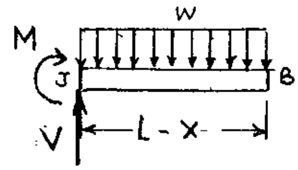
5.4 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

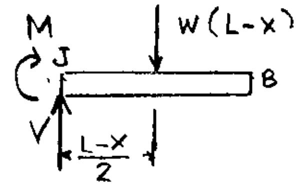






Use portion to the right of the section as the free body.





Replace distributed load by equivalent concentred load.

$+\uparrow \Sigma F_y = 0 \qquad V - w(L-x) = 0$

$$V = w(L-x) \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) = 0$

$$M = -\frac{w}{2}(L-x)^2 \blacktriangleleft$$

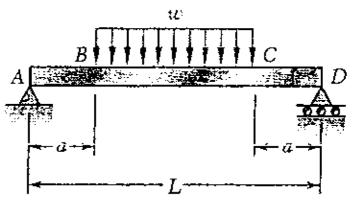
Largest negative bending moment occurs at $x = 0$.

$$M_{min} = -\frac{wL^2}{2} \blacktriangleleft$$

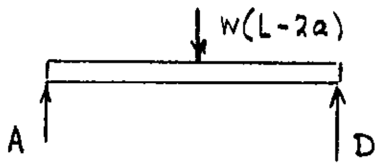
Thus, $\qquad |M|_{max} = \frac{wL^2}{2} \blacktriangleleft$

Problem 5.5

5.5 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

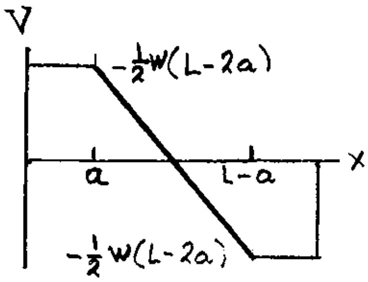


Calculate reactions after replacing distributed load by an equivalent concentrated load.

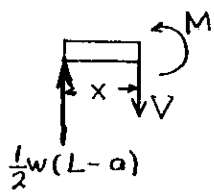


Reactions are

$$A = D = \tfrac{1}{2}w(L-2a)$$



From A to B $\quad 0 < x < a$

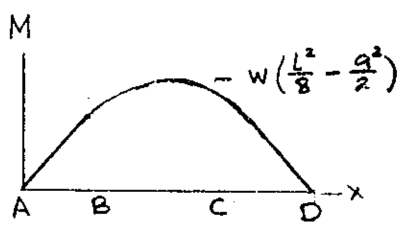


$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - V = 0$

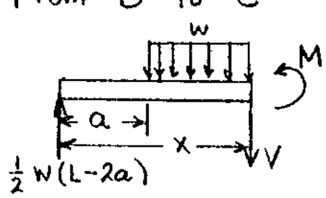
$$V = \tfrac{1}{2}w(L-2a)$$

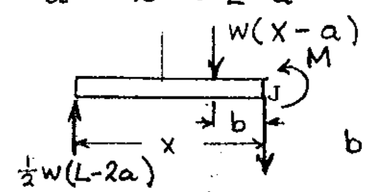
$\circlearrowleft \Sigma M = 0 \qquad -\frac{1}{2}w(L-2a) + M = 0$

$$M = \tfrac{1}{2}w(L-2a)x$$



From B to C $\quad a < x < L-a$





$b = \dfrac{x-a}{2}$

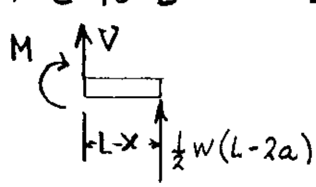
Place section cut at x. Replace distributed load by equiv. conc. load.

$+\uparrow \Sigma F_y = 0 \qquad \frac{1}{2}w(L-2a) - w(x-a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right)$

$\circlearrowleft M_J = 0 \qquad -\frac{1}{2}w(L-2a)x + w(x-a)\left(\frac{x-a}{2}\right) + M = 0$

$$M = \tfrac{1}{2}w\left[(L-2a)x - (x-a)^2\right]$$

From C to D $\quad L-a < x < L$



$+\uparrow \Sigma F_y = 0 \qquad V + \frac{1}{2}w(L-2a) = 0$

$$V = -\tfrac{w}{2}(L-2a)$$

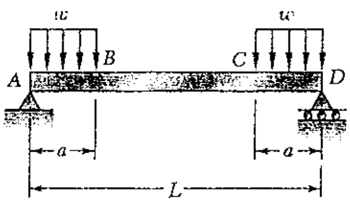
$\circlearrowleft \Sigma M_J = 0 \qquad -M + \frac{1}{2}w(L-2a)(L-x) = 0$

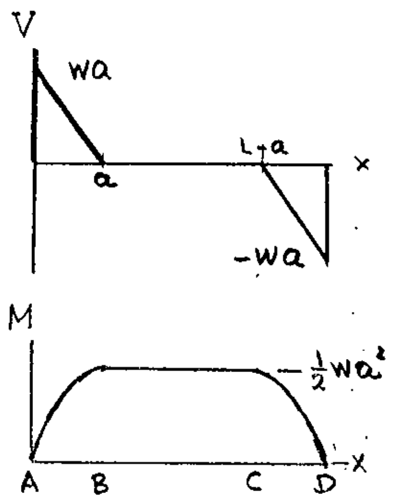
$$M = \tfrac{1}{2}w(L-2a)(L-x)$$

At $x = \frac{L}{2}$ $\qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$

Problem 5.6

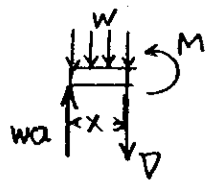
5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

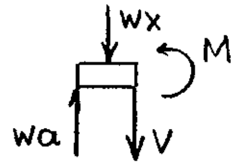




Reactions: $A = D = wa$

From A to B $\quad 0 < x < a$





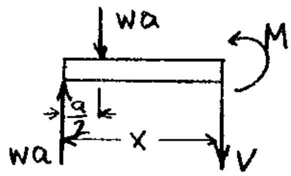
$+\uparrow \Sigma F_y = 0 \qquad wa - wx - V = 0$

$$V = w(a - x)$$

$\circlearrowleft \Sigma M_J = 0 \qquad -wax + (wx)\frac{x}{2} + M = 0$

$$M = w\left(ax - \frac{x^2}{2}\right)$$

From B to C $\quad a < x < L - a$



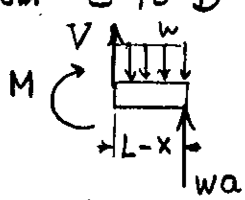
$\Sigma F_y = 0$

$wa - wa - V = 0$

$$V = 0$$

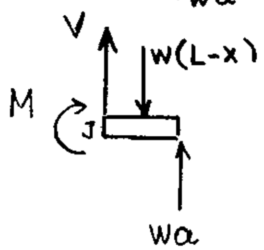
$\circlearrowleft \Sigma M_J = 0 \qquad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2$

From C to D $\quad L - a < x < L$



$+\uparrow \Sigma F_y = 0 \qquad V - w(L - x) + wa = 0$

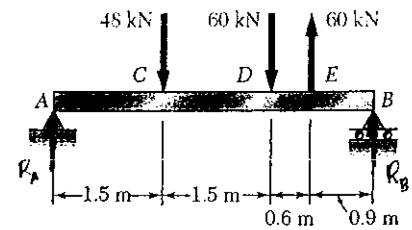
$$V = w(L - x - a)$$

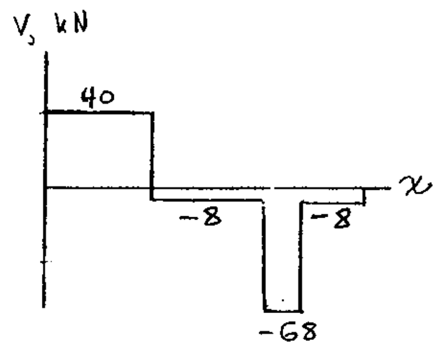


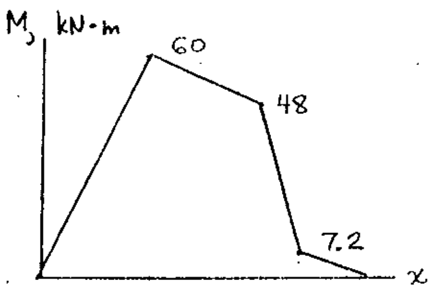
$\circlearrowleft \Sigma M_J = 0 \qquad -M - w(L-x)\left(\frac{L-x}{2}\right) + wa(L-x) = 0$

$$M = w\left[a(L-x) - \frac{1}{2}(L-x)^2\right]$$

Problem 5.7







5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

Reactions

$+\circlearrowleft \Sigma M_B = 0$

$-4.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40 \text{ kN}$

$+\circlearrowleft \Sigma M_A = 0$

$-(1.5)(48) - (3.0)(60) + (3.6)(60) + 4.5 R_B = 0$

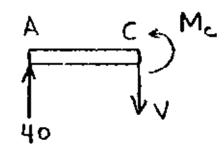
A to C	$V = 48$ kN
C to D	$V = -8$ kN
D to E	$V = -68$ kN
E to B	$V = -8$ kN

At A and B $\quad M = 0$

At C $\quad \circlearrowleft \Sigma M_C = 0$

$-(1.5)(40) + M_C = 0$

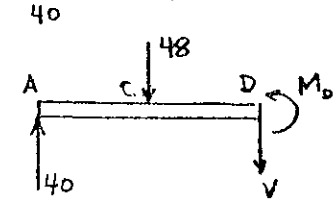
$M_C = 60 \text{ kN}\cdot\text{m}$



At D $\quad +\circlearrowleft \Sigma M_D = 0$

$-(3.0)(40) + (1.5)(48) + M_D = 0$

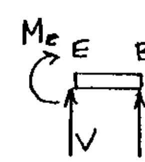
$M_D = 48 \text{ kN}\cdot\text{m}$



At E $\quad +\circlearrowleft \Sigma M_E = 0$

$-M_E + (0.9)(8) = 0$

$M_E = 7.2 \text{ kN}\cdot\text{m}$

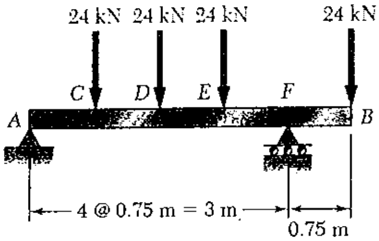


(a) $|V|_{max} = 68.0$ kN ◄

(b) $|M|_{max} = 60.0$ kN·m ◄

Problem 5.8

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reactions at A and F.

$+\circlearrowleft \Sigma M_F = 0$

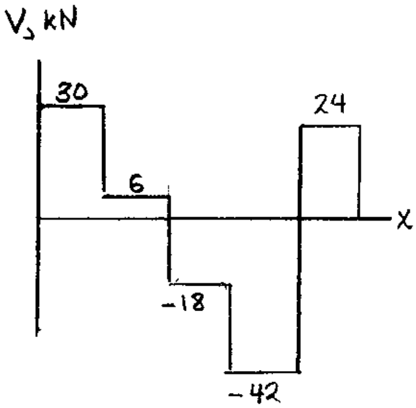
$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24) - (0.75)(24) = 0$

$R_A = 30 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

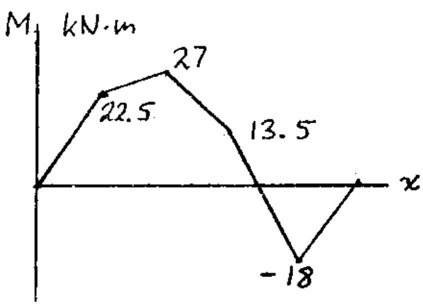
$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F - (3.75)(24) = 0$

$R_F = 66 \text{ kN} \uparrow$



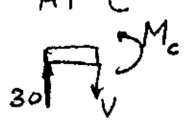
Shear diagram.

A to C	$V = 30$ kN
C to D	$V = 30 - 24 = 6$ kN
D to E	$V = 6 - 24 = -18$ kN
E to F	$V = -18 - 24 = -42$ kN
F to B	$V = -42 + 66 = 24$ kN



At A and B $\quad M_A = M_B = 0$

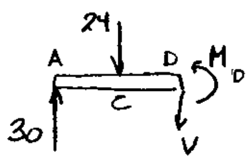
At C $\quad +\circlearrowleft \Sigma M_C = 0$



$-(0.75)(30) + M_C = 0$

$M_C = 22.5 \text{ kN}\cdot\text{m}$

At D

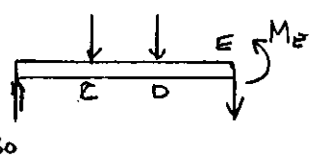


$+\circlearrowleft \Sigma M_D = 0$

$-(1.50)(30) + (0.75)(24) + M_D = 0$

$M_D = 27 \text{ kN}\cdot\text{m}$

At E

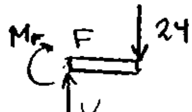


$+\circlearrowleft \Sigma M_E = 0$

$-(2.25)(30) + (1.50)(24) + (0.75)(24) + M_E = 0$

$M_E = 13.5 \text{ kN}\cdot\text{m}$

At F



$+\circlearrowleft \Sigma M_F = 0$

$-M_F - (0.75)(24) = 0$

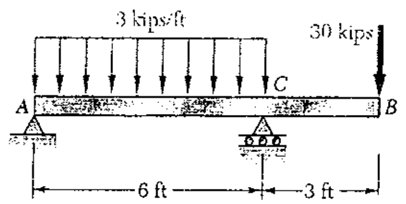
$M_F = -18 \text{ kN}\cdot\text{m}$

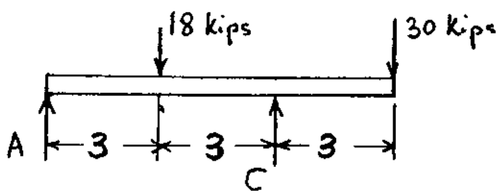
(a) $|V|_{max} = 42.0 \text{ kN}$ ◄

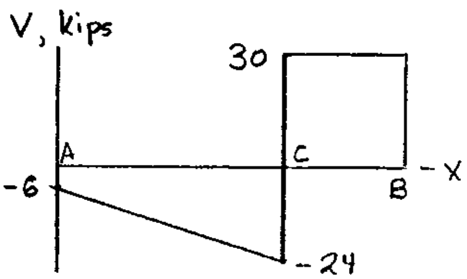
(b) $|M|_{max} = 27.0 \text{ kN}\cdot\text{m}$ ◄

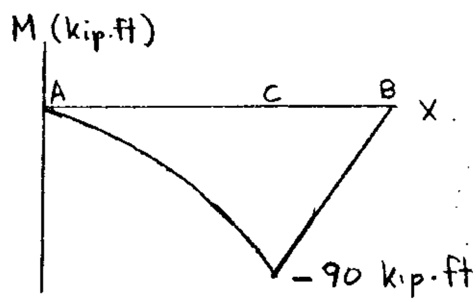
Problem 5.9

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.









Reactions

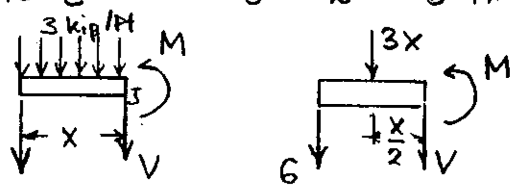
$\circlearrowleft \Sigma M_C = 0 \qquad -6A + (3)(18) - (3)(30) = 0$

$A = -6 \text{ kips} \quad \text{i.e. } 6 \text{ kips} \downarrow$

$\circlearrowleft \Sigma M_A = 0 \qquad 6C - (3)(18) - (9)(30) = 0$

$C = 54 \text{ kips} \uparrow$

A to C $\qquad 0 < x < 6$ ft.



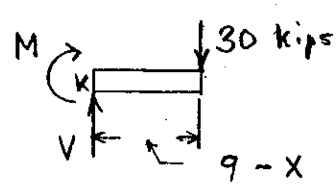
$+\uparrow \Sigma F_y = 0 \qquad -6 - 3x - V = 0$

$V = -6 - 3x$ kips.

$\circlearrowleft \Sigma M_J = 0 \qquad -6x - (3x)(\frac{x}{2}) - M = 0$

$M = -6x - 1.5x^2$ kip·ft

C to B $\qquad 6 \text{ ft} < x < 9$ ft



$+\uparrow \Sigma F_y = 0 \qquad V - 30 = 0$

$V = 30$ kips

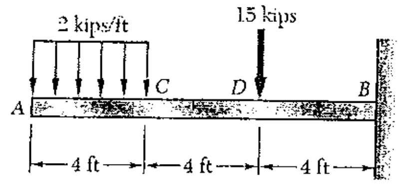
$\circlearrowleft \Sigma M_K = 0 \qquad -M - (9 - x)(30) = 0$

$M = 30x - 270$ kip·ft

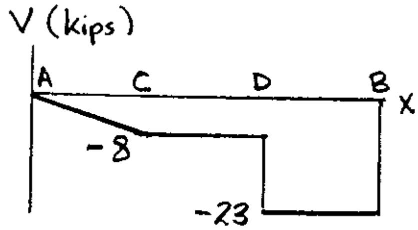
From the diagrams $\qquad$ (a) $|V|_{max} = 30.0$ kips ◄

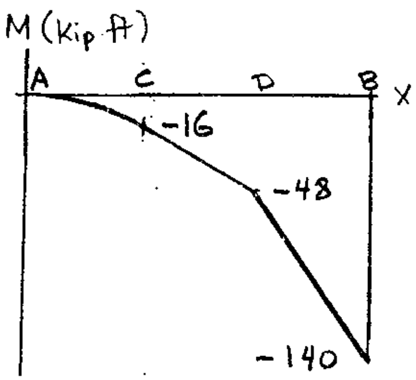
(b) $|M|_{max} = 90.0$ kip·ft ◄

Problem 5.10

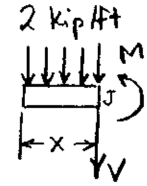


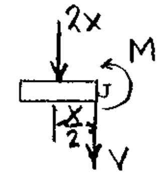
5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.





A to C
$0 < x < 4$ ft





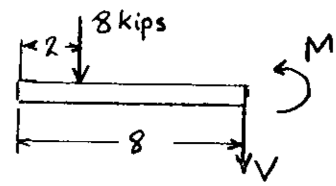
$+\uparrow \Sigma F_y = 0 \qquad -V - 2x = 0 \qquad V = -2x$ kips

$+\circlearrowleft \Sigma M_J = 0 \qquad M + (2x)\left(\frac{x}{2}\right) = 0$

$M = -x$ kip·ft

At C $\qquad V = -8$ kips $\qquad M = -16$ kip·ft

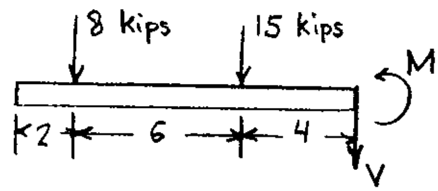
At D^-



$+\uparrow \Sigma F_y = 0 \qquad -8 - V = 0 \qquad V = -8$ kips

$+\circlearrowleft \Sigma M_D = 0 \qquad (6)(8) - M = 0 \qquad M = -48$ kip·ft

At B^-



$+\uparrow \Sigma F_y = 0 \qquad -8 - 15 - V = 0$

$V = -23$ kips

$+\circlearrowleft \Sigma M_B = 0 \qquad -(10)(8) - (4)(15) - M = 0$

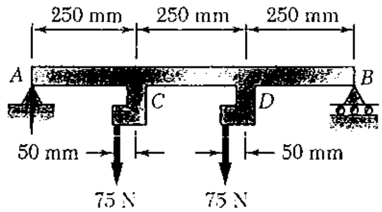
$M = -140$ kip·ft

From the diagrams $\quad$ (a) $|V|_{max} = 23.0$ kips ◄

(b) $|M|_{max} = 140.0$ kip·ft ◄

Problem 5.11

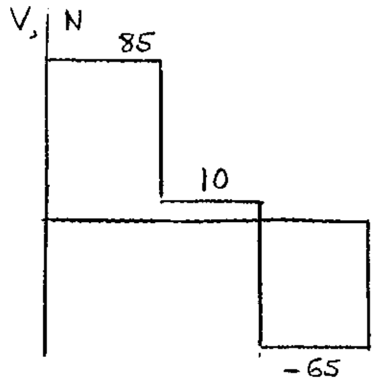
5.11 and *5.12* Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reaction at A. $+\circlearrowleft M_B = 0$

$-0.750\,R_A + (0.550)(75) + (0.300)(75) = 0$

$R_A = 85\text{ N}\uparrow$ Also $R_B = 65\text{ N}\uparrow$

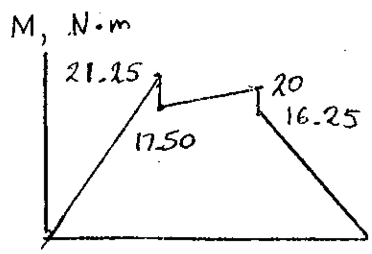


A to C $\quad V = 85\text{ N}$

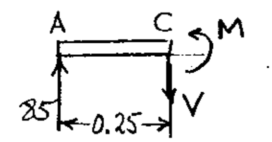
C to D $\quad V = 10\text{ N}$

D to B $\quad V = -65\text{ N}$

At A and B $\quad M = 0$



Just to the left of C

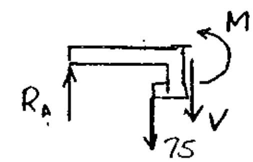


$\circlearrowleft \Sigma M_C = 0$

$-(0.25)(85) + M = 0$

$M = 21.25\text{ N}\cdot\text{m}$

Just to the right of C

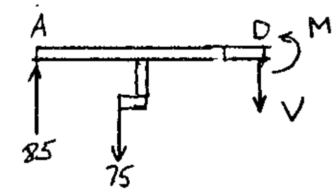


$+\circlearrowleft \Sigma M_C = 0$

$-(0.25)(85) + (0.050)(75) + M = 0$

$M = 17.50\text{ N}\cdot\text{m}$

Just to the left of D

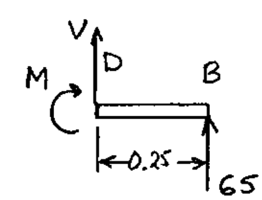


$+\circlearrowleft \Sigma M_D = 0$

$-(0.50)(85) + (0.300)(75) + M = 0$

$M = 20\text{ N}\cdot\text{m}$

Just to the right of D



$+\circlearrowleft \Sigma M_D = 0$

$-M + (0.25)(65) = 0$

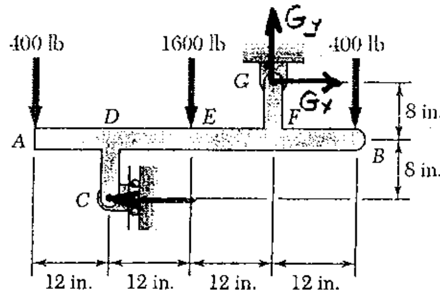
$M = 16.25\text{ kN}$

(a) $|V|_{max} = 85.0\text{ N}$ ◀

(b) $|M|_{max} = 21.25\text{ N}\cdot\text{m}$ ◀

Problem 5.12

5.11 and 5.12 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



$\circlearrowleft \sum M_G = 0$

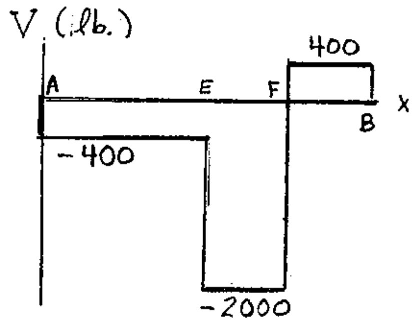
$-16\,C + (36)(400) + (12)(1600) - (12)(400) = 0$

$C = 1800$ lb.

$\xrightarrow{+} \sum F_x = 0 \qquad -C + G_x = 0 \qquad G_x = 1800$ lb.

$+\uparrow \sum F_y = 0 \qquad -400 - 1600 + G_y - 400 = 0$

$G_y = 2400$ lb.

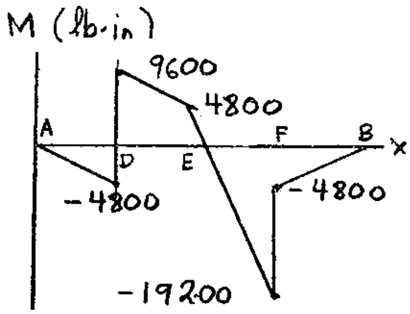


A to E $\qquad V = -400$ lb.

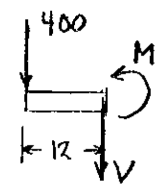
E to F $\qquad V = -2000$ lb.

F to B $\qquad V = 400$ lb.

At A and B $\qquad M = 0$



At D^-

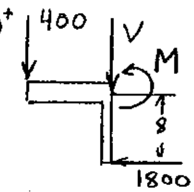


$\circlearrowleft \sum M_D = 0$

$(12)(400) + M = 0$

$M = -4800$ lb·in.

At D^+

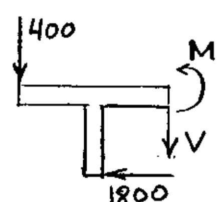


$\circlearrowleft \sum M_D = 0$

$(12)(400) - (8)(1800) + M = 0$

$M = 9600$ lb·in.

At E



$\circlearrowleft \sum M_E = 0$

$(24)(400) - (8)(1800) + M = 0$

$M = 4800$ lb·in

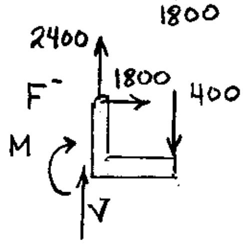
(a) Maximum $|V|$

$= 2000$ lb ◄

(b) Maximum $|M|$

$= 19200$ lb·in. ◄

At F^-

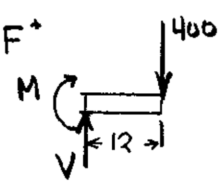


$\circlearrowleft \sum M_F = 0$

$-M - (8)(1800) - (12)(400) = 0$

$M = -19200$ lb·in

At F^+



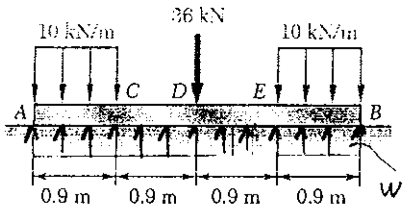
$\sum M_F = 0$

$-M - (12)(400) = 0$

$M = -4800$ lb·in.

Problem 5.13

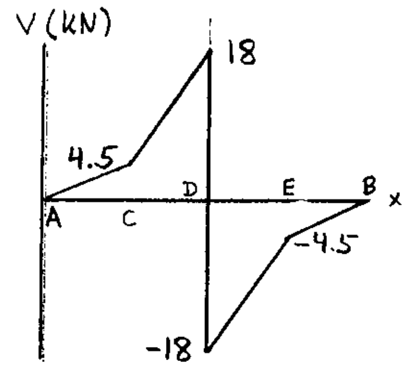
5.13 and *5.14* Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam AB and determine the maximum absolute value of (a) of the shear, (b) of the bending moment.



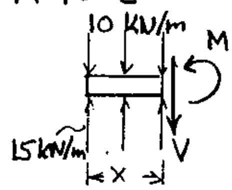
Over whole beam $+\uparrow \Sigma F_y = 0$

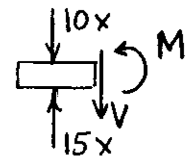
$$3.6\,w - (0.9)(10) - 36 - (0.9)(10) = 0$$

$$w = 15 \text{ kN/m}$$



A to C $\qquad 0 < x < 0.9$ m



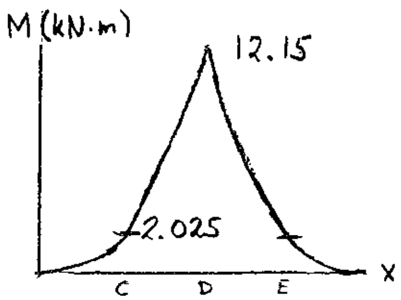


$+\uparrow \Sigma F_y = 0 \qquad 15x - 10x - V = 0$

$$V = 5x$$

$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\frac{x}{2} + (10x)\frac{x}{2} + M = 0$

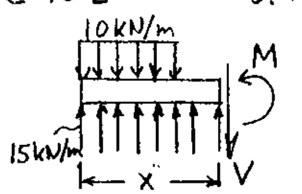
$$M = 2.5x^2$$

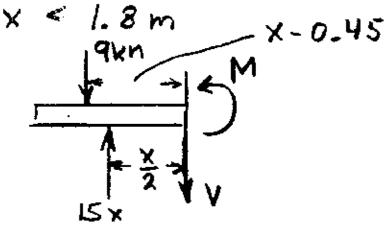


At $x = C \qquad V = 4.5$ kN

$\qquad\qquad\quad M = 2.025$ kN·m

C to D $\qquad 0.9 \text{ m} < x < 1.8 \text{ m}$





(a) Maximum $|V|$
$= 18$ kN

$+\uparrow \Sigma F_y = 0 \qquad 15x - 9 - V = 0$

$$V = 15x - 9$$

(b) Maximum $|M|$
$= 12.15$ kN·m

$\circlearrowleft \Sigma M_J = 0 \qquad -(15x)\left(\frac{x}{2}\right) + 9(x - 0.45) + M = 0$

$$M = 7.5x^2 - 9x + 4.05 = 0$$

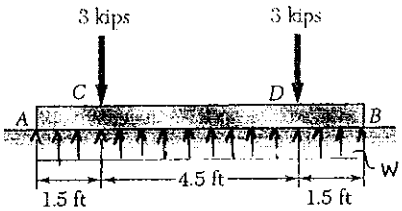
At D^- $\qquad V = 18$ kN

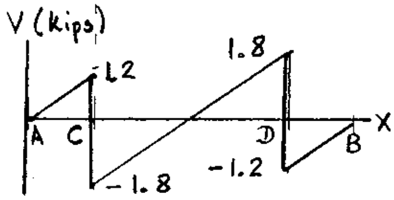
$\qquad\qquad M = 12.15$ kN·m

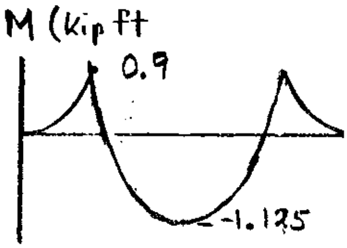
D to B $\qquad$ Use symmetry to calculate the shear and bending moment.

Problem 5.14

5.13 and ***5.14*** Assuming that the reaction of the ground to be uniformly distributed, draw the shear and bending-moment diagrams for the beam *AB* and determine the maximum absolute value of (*a*) of the shear, (*b*) of the bending moment.





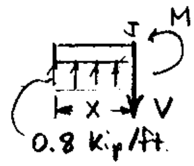


Over the whole beam

$+\uparrow \Sigma F_y = 0 \qquad 7.5w - 3 - 3 = 0$

$w = 0.8 \text{ kip/ft}$

A to C $\qquad 0 < x < 1.5 \text{ ft}$



$+\uparrow \Sigma F_y = 0 \qquad 0.8x - V = 0$

$V = 0.8x$

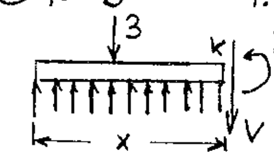
$\circlearrowleft \Sigma M_J = 0$

$-(0.8x)(\frac{x}{2}) + M = 0$

$M = 0.4x^2$

At C^- $\qquad V = 1.2 \text{ kips}, \qquad M = 0.9 \text{ kip·ft}$

C to D $\qquad 1.5 \text{ ft} < x < 6 \text{ ft}$



$+\uparrow \Sigma F_y = 0$

$0.8x - 3 - V = 0$

$V = 0.8x - 3$

$\circlearrowleft \Sigma M_K = 0 \qquad -(0.8x)(\frac{x}{2}) + 3(x - 1.5) + M = 0$

$M = 0.4x^2 - 3x + 4.5$

At the center of the beam $\qquad x = 3.75 \text{ ft}$

$V = 0, \qquad M = -1.125 \text{ kip ft}$

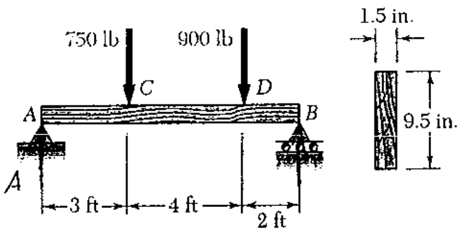
At C^+ $\qquad V = -1.8 \text{ kip} \qquad M = 0.9 \text{ kip ft}$

(a) Maximum $|V| = 1.8$ kips ◄

(b) Maximum $|M| = 1.125$ kip·ft ◄

Problem 5.15

5.15 and ***5.16*** For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



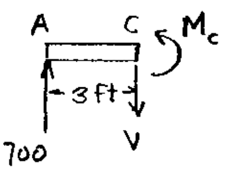
Reaction at A

$$\circlearrowleft \Sigma M_B = 0$$

$$-9A + (6)(750) + (2)(900) = 0$$

$$A = 700 \text{ lb.}$$

Using free body. A to C.



$$\circlearrowleft \Sigma M_C = 0 \qquad -(700)(3) + M_c = 0$$

$$M_c = 2100 \text{ lb}\cdot\text{ft} = 25.2\times10^3 \text{ lb}\cdot\text{in} = 25.2 \text{ kip}\cdot\text{in}$$

For the cross section $I = \frac{1}{12}(1.5)(9.5)^3 = 107.172 \text{ in}^4$

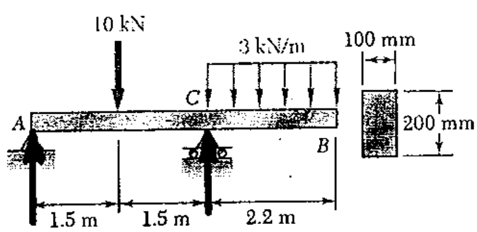
$$c = 4.75 \text{ in.}$$

Maximum normal stress due to bending.

$$\sigma = \frac{Mc}{I} = \frac{(25.2)(4.75)}{107.172} \qquad \sigma = 1.117 \text{ ksi}$$

Problem 5.16

5.15 and ***5.16*** For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

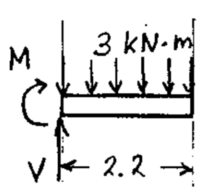


Using CB as a free body

$$\circlearrowleft \Sigma M_C = 0$$

$$-M + (2.2)(3\times10^3)(1.1) = 0$$

$$M = 7.26\times10^3 \text{ N}\cdot\text{m}$$



Section modulus for rectangle

$$S = \frac{1}{6}bh^2$$

$$= \frac{1}{6}(100)(200)^2 = 666.7\times10^3 \text{ mm}^3$$

$$= 666.7\times10^{-6} \text{ m}^3$$

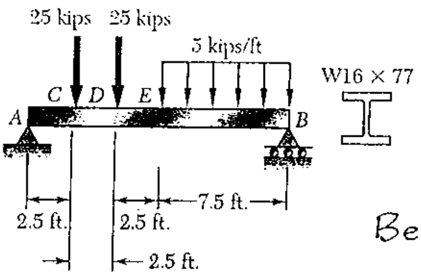
Normal stress

$$\sigma = \frac{M}{S} = \frac{7.26\times10^3}{666.7\times10^{-6}} = 10.89\times10^6 \text{ Pa}$$

$$\sigma = 10.89 \text{ MPa}$$

Problem 5.17

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



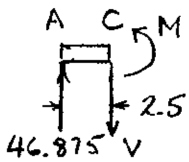
Reaction at A. $\circlearrowleft \Sigma M_B = 0$

$$-15.0 R_A + (12.5)(25) + (10.0)(25) + (3.75)(7.5)(5) = 0 \qquad R_A = 46.875 \text{ kips}$$

Bending moment at C. $\circlearrowleft \Sigma M_C = 0$

$$-(2.5)(46.875) + M = 0$$

$$M = 117.1875 \text{ kip}\cdot\text{ft} = 1.40625 \times 10^3 \text{ kip}\cdot\text{in}$$



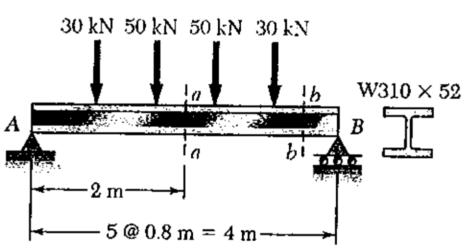
For W16×77 rolled steel section $\qquad S = 134 \text{ in}^3$

Normal stress at C

$$\sigma = \frac{M}{S} = \frac{1.40625 \times 10^3}{134} = \qquad \sigma = 10.49 \text{ ksi}$$

Problem 5.18

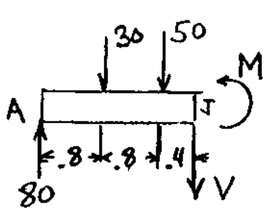
5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.



Reactions: $\qquad$ By symmetry $\qquad A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80 \text{ kN}$

Using left half of beam as free body



$$\circlearrowleft \Sigma M_J = 0$$

$$-(80)(2) + (30)(1.2) + (50)(0.4) + M = 0$$

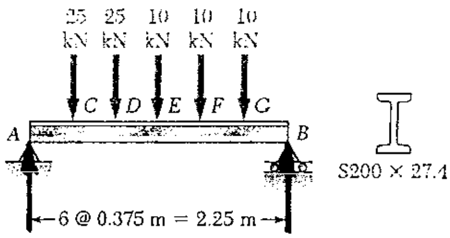
$$M = 104 \text{ kN}\cdot\text{m} = 104 \times 10^3 \text{ N}\cdot\text{m}$$

For W310 × 52 $\qquad S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$

Normal stress $\qquad \sigma = \dfrac{M}{S} = \dfrac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa} = 139.0 \text{ MPa}$

Problem 5.19

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

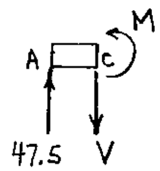


Use entire beam as free body

$\circlearrowleft + \; \Sigma M_B = 0$

$2.25 A - (1.875)(25) - (1.5)(25) - (1.125)(10) - (0.75)(10) - (0.375)(10) = 0$

$A = 47.5 \text{ kN}$



Use portion AC as free body

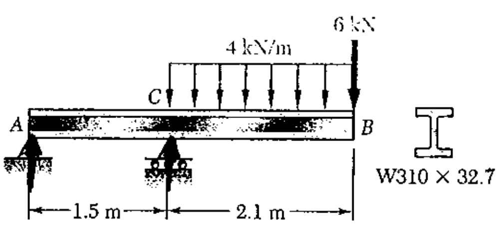
$-(0.375)(47.5) + M = 0 \qquad M = 17.8125 \text{ kN}\cdot\text{m}$

For S 200 × 27.4 $\qquad S = 235 \times 10^3 \text{ mm}^3 = 235 \times 10^{-6} \text{ m}^3$

Normal stress $\qquad \sigma = \dfrac{M}{S} = \dfrac{17.8125 \times 10^3}{235 \times 10^{-6}} = 75.8 \times 10^6 \text{ Pa} = 75.8 \text{ MPa}$ ◄

Problem 5.20

5.19 and 5.20 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

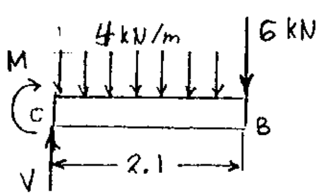


Use portion CB as free body.

$\circlearrowleft + \; \Sigma M_C = 0$

$-M + (4)(2.1)(1.05) + (6)(2.1) = 0$

$M = -21.42 \text{ kN}\cdot\text{m}$



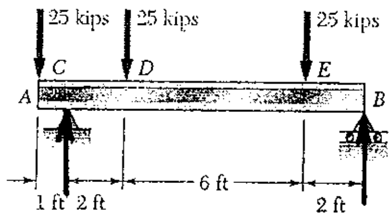
For W 310 × 32.7 $\qquad S = 415 \times 10^3 \text{ mm}^3 = 415 \times 10^{-6} \text{ m}^3$

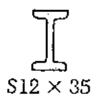
Normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{21.42 \times 10^3}{415 \times 10^{-6}} = 51.6 \times 10^6 \text{ Pa}$

$= 51.6 \text{ MPa}$ ◄

Problem 5.21

5.21 and 5.22 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.





$\circlearrowleft + \sum M_B = 0$

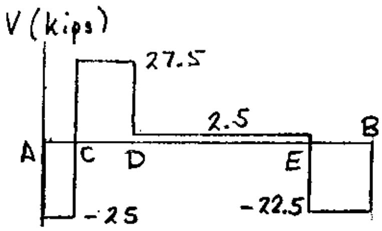
$(11)(25) - 10C + (8)(25) + (2)(25) = 0$

$C = 52.5$ kips

$\circlearrowleft + \sum M_C = 0$

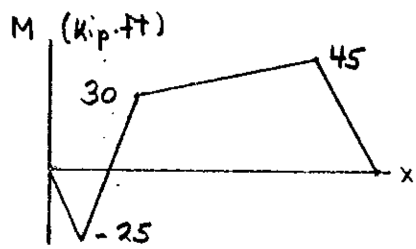
$(1)(25) - (2)(25) - (8)(25) + 10B = 0$

$B = 22.5$ kips



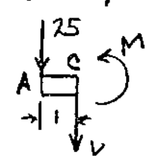
Shear

A to C^-	$V = -25$ kips
C^+ to D^-	$V = 27.5$ kips
D^+ to E^-	$V = 2.5$ kips
E^+ to B	$V = -22.5$ kips



Bending moments

At C

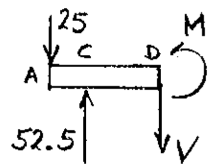


$\circlearrowleft + \sum M_C = 0$

$(1)(25) + M = 0$

$M = -25$ kip·ft

At D

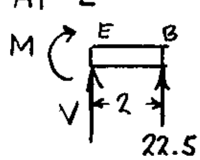


$\circlearrowleft + \sum M_D = 0$

$(3)(25) - (2)(52.5) + M = 0$

$M = 30$ kip·ft

At E



$\circlearrowleft + \sum M_E = 0 \qquad -M + (2)(22.5) = 0 \qquad M = 45$ kip·ft

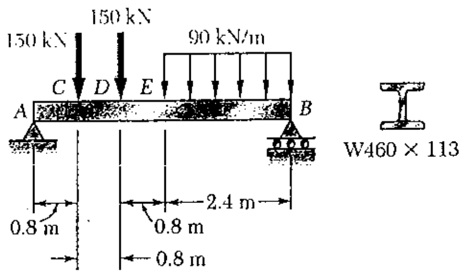
max $|M| = 45$ kip·ft $= 540$ kip·in

For S12 × 35 rolled steel section $\qquad S = 38.2$ in^3

Normal stress $\qquad \sigma = \dfrac{|M|}{S} = \dfrac{540}{38.2} = 14.14$ ksi

Problem 5.22

5.21 and 5.22 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at A $\quad +\circlearrowleft \Sigma M_B = 0$

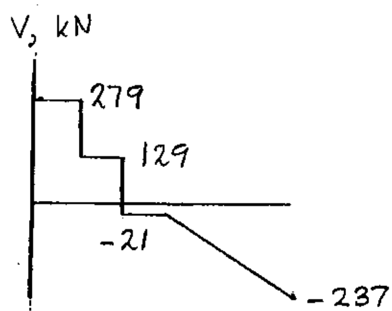
$$-4.8 R_A + (4.0)(150) + (3.2)(150) + (1.2)(2.4)(90) = 0$$

$$R_A = 279 \text{ kN}$$

Reaction at B $\quad +\circlearrowleft \Sigma M_A = 0$

$$-(0.8)(150) - (1.6)(150) - (3.6)(2.4)90 + 4.8 R_B = 0$$

$$R_B = 237 \text{ kN}$$

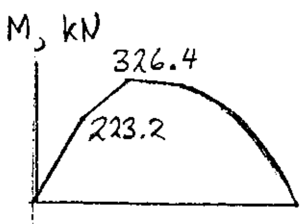


A to C $\quad V = 279$ kN

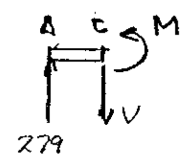
C to D $\quad V = 279 - 150 = 129$ kN

D to E $\quad V = 279 - 150 - 150 = -21$ kN

At B $\quad V = -237$ kN



At A and B $\quad M = 0$



At C $\quad +\circlearrowleft \Sigma M_C = 0$

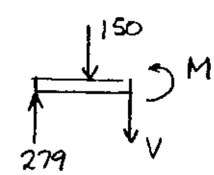
$$-(0.8)(279) + M = 0$$

$$M = 223.2 \text{ kN·m}$$

From diagram

$M_{max} = 326.4$ kN·m

$= 326.4 \times 10^3$ N·m



At D $\quad +\circlearrowleft M_D = 0$

$$-(1.6)(279) + (0.8)(150) + M = 0$$

$$M = 326.4 \text{ kN·m}$$

At E $\quad +\circlearrowleft M_E = 0$

$$-(2.4)(279) + (1.6)(150) + (0.8)(150) + M = 0$$

$$M = 309.6 \text{ kN·m}$$

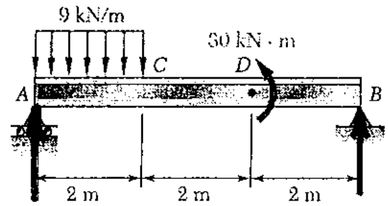
For W 460×113 rolled steel section $\quad S = 2400 \times 10^3 \text{ mm}^3 = 2400 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma_m = \dfrac{M_{max}}{S} = \dfrac{326.4 \times 10^3}{2400 \times 10^{-6}} = 136 \times 10^6$ Pa

$\sigma_m = 136.0$ MPa ◀

Problem 5.23

5.23 and 5.24 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



W200 × 22.5

$\circlearrowleft + \sum M_B = 0$

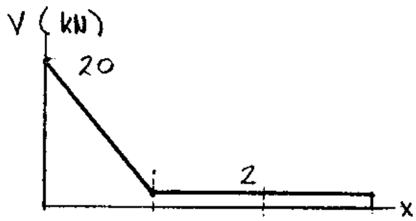
$-6A + (2)(9)(5) + 30 = 0$

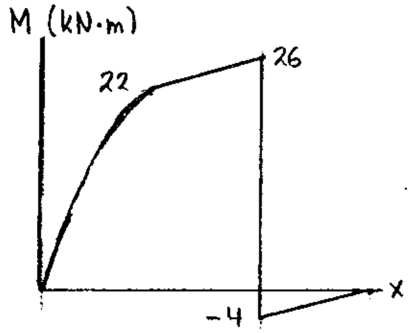
$A = 20 \text{ kN}$

$\circlearrowleft + \sum M_A = 0$

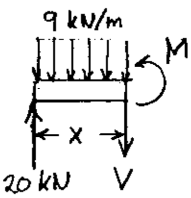
$-(2)(9)(1) + 30 + 6B = 0$

$B = -2 \text{ kN}$ i.e $2 \text{ kN} \downarrow$





A to C $\quad 0 < x < 2 \text{ m}$



$+\uparrow \sum F_y = 0 \qquad 20 - 9x - V = 0$

$V = 20 - 9x$

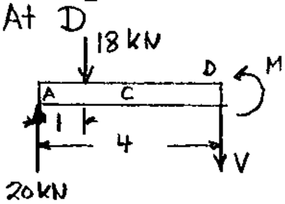
$\circlearrowleft \sum M_J = 0$

$-20x + (9x)\frac{x}{2} + M = 0$

$M = 20x - 4.5x^2$

At C $\quad V = 2 \text{ kN} \quad M = 22 \text{ kN·m}$

At D^-



$+\uparrow \sum F_y = 0$

$20 - 18 - V = 0$

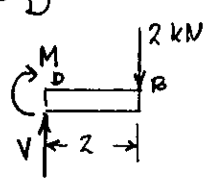
$V = 2 \text{ kN}$

$\circlearrowleft \sum M_D = 0$

$-(4)(20) + (3)(18) + M = 0$

$M = 26 \text{ kN·m}$

At D^+



$+\uparrow \sum F_y = 0 \qquad V - 2 = 0$

$V = 2 \text{ kN}$

$\circlearrowleft \sum M_D = 0$

$-M - (2)(2) = 0$

$M = -4 \text{ kN·m}$

$\max |M| = 26 \text{ kN·m} = 26 \times 10^3 \text{ N·m}$

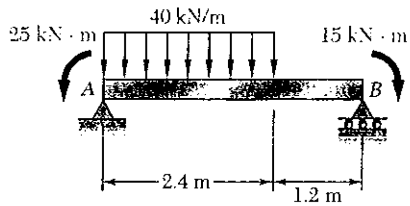
For rolled steel section W $200 \times 22.5 \qquad S = 194 \times 10^3 \text{ mm}^3$

$= 194 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{26 \times 10^3}{194 \times 10^{-6}} = 134.0 \times 10^6 \text{ Pa} = 134.0 \text{ MPa}$ ◄

Problem 5.24

5.23 and 5.24 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



W310 × 38.7

Reaction at A: $\circlearrowleft \Sigma M_B = 0$

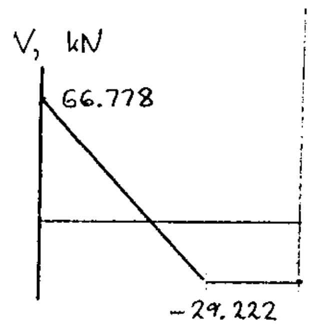
$-3.6 R_A + 2 \quad (2.4)(2.4)(40) - 15 = 0$

$R_A = 66.778 \text{ kN}$

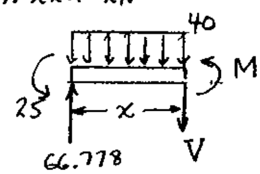
Reaction at B $\quad +\circlearrowright M_A = 0$

$25 - (1.2)(2.4)(40) - 15 + 3.6 R_B = 0$

$R_B = 29.222 \text{ kN}$



$0 < x \leq 2.4 \text{ m}$

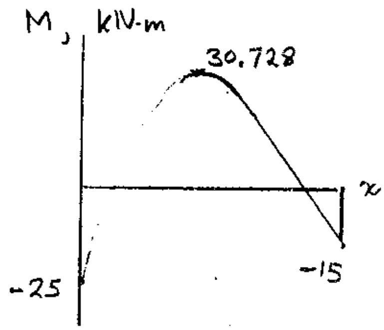


$+\uparrow \Sigma F_y = 0$

$66.778 - 40x - V = 0$

$V = 66.778 - 40x$

$V = 0 \quad \text{at} \quad x = 1.6944 \text{ m}$



$+\circlearrowright \Sigma M = 0$

$25 + (40x)\frac{x}{2} - 66.778x + M = 0$

$M = -20x^2 + 66.778x - 25$

$M = -(20)(1.6944)^2 + (66.778)(1.6944) - 25$
$= 30.728 \text{ kN·m} \quad \text{at} \quad x = 1.6944 \text{ m}$

$2.4 \leq x < 3.6 \qquad V = -29.222 \text{ kN}$

$M = 29.222(3.6 - x) - 15$

$M = 20.067 \text{ kN·m} \quad \text{at} \quad x = 2.4 \text{ m}$

$M_{max} = 30.728 \text{ kN·m} = 30.728 \times 10^3 \text{ N·m}$

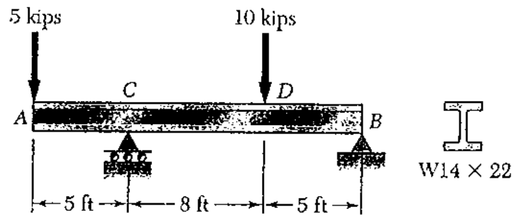
For W310 × 38.7 rolled steel section $S = 549 \times 10^3 \text{ mm}^3$
$= 549 \times 10^{-6} \text{ m}^3$

$$\sigma_m = \frac{M_{max}}{S} = \frac{30.728 \times 10^3}{549 \times 10^{-6}} = 55.97 \times 10^6 \text{ Pa}$$

56.0 MPa ◄

Problem 5.25

5.25 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Reaction at C $\quad +\circlearrowleft M_B = 0$

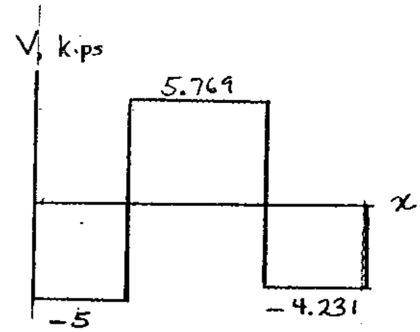
$(18)(5) - 13C + (5)(10) = 0$

$C = 10.769$ kips

Reaction at B $\quad +\circlearrowleft M_C = 0$

$(5)(5) - (8)(10) + 13B = 0$

$B = 4.231$ kips



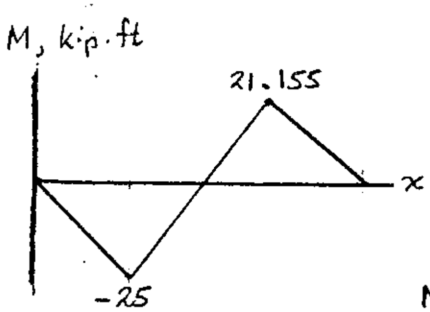
Shear diagram.

A to C^- $\quad V = -5$ kips

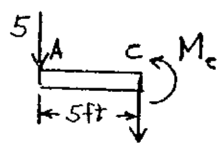
C^+ to D^- $\quad V = -5 + 10.769 = 5.769$ kips

D^+ to B $\quad V = 5.769 - 10 = -4.231$ kips

At A and B $\quad M = 0$



At C

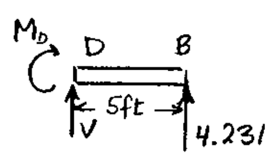


$+\circlearrowleft M_C = 0$

$(5)(5) + M_C = 0$

$M_C = -25$ kip·ft

At D



$+\circlearrowleft \Sigma M_D = 0$

$-M_D + (5)(4.231)$

$M_D = 21.155$ kip·ft

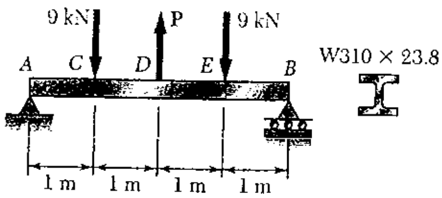
$|M|_{max}$ occurs at C $\qquad |M|_{max} = 25$ kip·ft $= 300$ kip·in.

For W 14 × 22 rolled steel section $\quad S = 29.0\ \text{in}^3$

Normal stress $\qquad \sigma = \dfrac{M}{S} = \dfrac{300}{29.0} = 10.34$ ksi

Problem 5.26

5.26 Knowing that $P = 10$ kN, draw the shear and bending-moment diagrams for beam AB and determine the maximum normal stress due to bending.



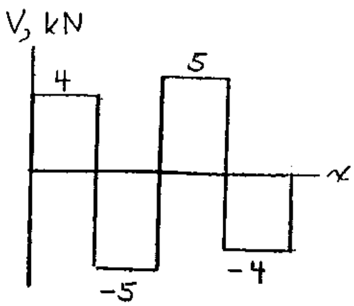
By symmetry the reactions at A and B are equal.

$A = B$

$+\uparrow \Sigma F_y = 0: \quad A + B - 9 + 10 - 9 = 0$

$A = B = 4 \text{ kN}$

Shear diagram.



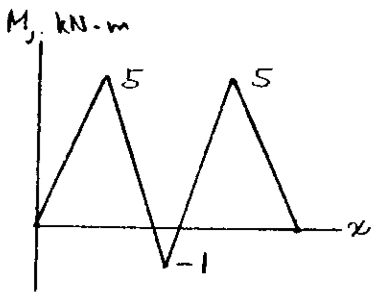
A to C $\quad V = 4 \text{ kN}$

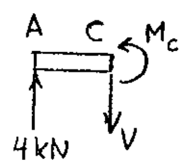
C to D $\quad V = 4 - 9 = -5 \text{ kN}$

D to E $\quad V = -5 + 10 = 5 \text{ kN}$

E to B $\quad V = 5 - 9 = -4 \text{ kN}$

Bending moments. $\quad M_A = M_B = 0$

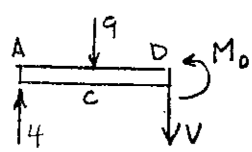




$+\circlearrowleft \Sigma M_C = 0$

$-(1)(4) + M_C = 0$

$M_C = 4 \text{ kN·m}$



$+\circlearrowleft \Sigma M_D = 0$

$-(2)(4) + (1)(9) + M_D = 0$

$M_D = -1 \text{ kN·m}$

By symmetry $\quad M_E = M_C = 4 \text{ kN·m}$

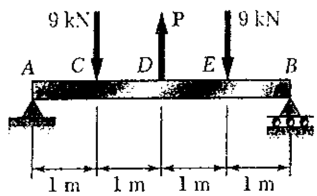
$|M|_{max} = M_C = 4 \text{ kN·m} = 4 \times 10^3 \text{ N·m}$

For W310 × 23.8 $\quad S = 280 \times 10^3 \text{ mm}^3 = 280 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{4 \times 10^3}{280 \times 10^{-6}} = 14.29 \times 10^6 \text{ Pa} \qquad 14.29 \text{ MPa}$

Problem 5.27

5.27 Determine (*a*) the magnitude of the upward force **P** for which the maximum absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (*Hint:* Draw the bending-moment diagram and the equate the absolute values of the largest positive and negative bending moments obtained..)



W310 × 23.8

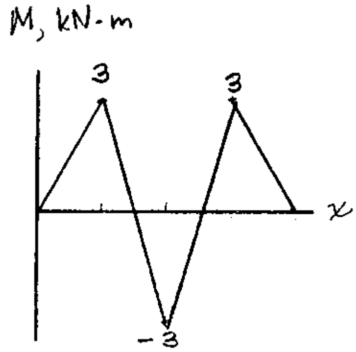
By symmetry the reactions at A and B are equal.

$A = B$

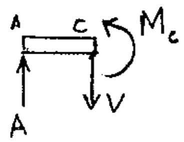
$+\uparrow \Sigma F_y = 0: \quad A - 9 + P - 9 + B = 0$

$A = B = 9 - \frac{1}{2}P \qquad P = 18 - 2A$

Also, by symmetry bending moments $M_C = M_E$.



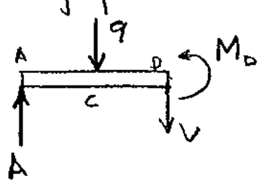
Using portion AC as a free body



$+\circlearrowleft \Sigma M_c = 0$

$-1A + M_c = 0 \qquad M_c = 1A$

Using portion ACD as a free body



$+\circlearrowleft \Sigma M_D = 0$

$-2A + (1)(9) + M_D = 0$

$M_D = 2A - 9$

Equate $\quad M_c = -M_D$

$1A = 9 - 2A \qquad A = 3 \text{ kN}$

Then $\quad P = 18 - (2)(3) = 12 \text{ kN} \qquad$ (a) 12.00 kN ◄

$M_c = 3 \text{ kN}\cdot\text{m}$

$M_D = (2)(3) - 9 = -3 \text{ kN}\cdot\text{m}$

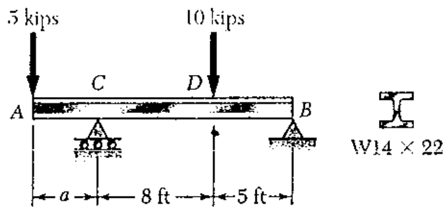
$|M|_{max} = 3 \text{ kN}\cdot\text{m} = 3\times10^3 \text{ N}\cdot\text{m}$

For W310 × 23.4 rolled steel section $\quad S = 280\times10^3 \text{ mm}^3 = 280\times10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{3\times10^3}{280\times10^{-6}} = 10.71\times10^6 \text{ Pa} \qquad$ (b) 10.71 MPa ◄

Problem 5.28

5.28 Determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27) (*Hint:* Draw the bending-moment diagram and the equate the absolute values of the largest positive and negative bending moments obtained..)

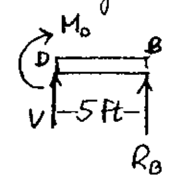


Reaction at B. $+\circlearrowleft \Sigma M_C = 0$

$$5a - (8)(10) + 13 R_B = 0$$

$$R_B = \frac{1}{13}(80 - 5a)$$

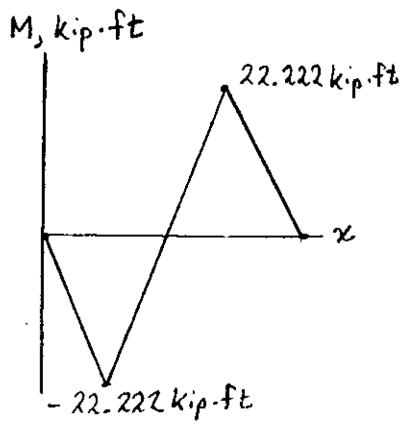
Bending moment at D.



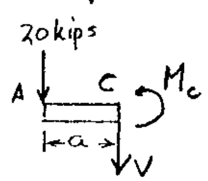
$+\circlearrowleft \Sigma M_D = 0$

$$-M_D + 5R_B = 0$$

$$M_D = 5R_B = \frac{5}{13}(80 - 5a)$$



Bending moment at C



$+\circlearrowleft M_C = 0$

$$5a + M_C = 0$$

$$M_C = -5a$$

Equate $-M_C = M_D$

$$5a = \frac{5}{13}(80 - 5a)$$

$$a = 4.4444 \text{ ft} \qquad \text{(a)} \quad a = 4.44 \text{ ft}$$

Then $-M_C = M_D = (5)(4.4444) = 22.222$ kip ft

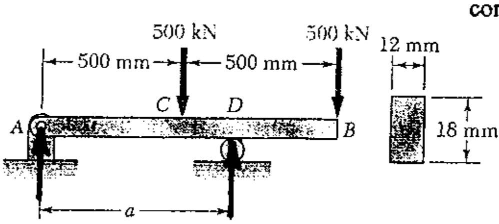
$|M|_{max} = 22.222$ kip·ft $= 266.67$ kip·in

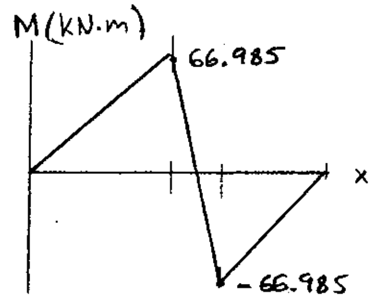
For W 14 × 22 rolled steel section $S = 29.0 \text{ in}^3$

Normal stress $\sigma = \frac{M}{S} = \frac{266.67}{29.0} = 9.20$ ksi (b) 9.20 ksi

Problem 5.29

5.29 For the beam and loading shown, determine (*a*) the distance *a* for which the absolute value of the bending moment in the beam is as small as possible, (*b*) the corresponding maximum normal stress due to bending. (See hint of Prob. 5.27)





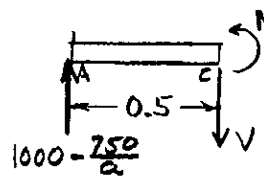
Reaction at A $\quad \circlearrowleft \Sigma M_D = 0$

$$-Aa + (500)(a - 0.5) - 500(1 - a) = 0$$

$$Aa = 1000a - 750$$

$$A = 1000 - \frac{750}{a}$$

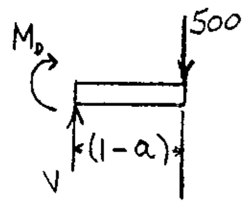
Bending moment at C $\quad \circlearrowleft \Sigma M_C = 0$



$$-(0.5)\left(1000 - \frac{750}{a}\right) + M_C = 0$$

$$M_C = 500 - \frac{375}{a}$$

Bending moment at D $\quad \circlearrowleft \Sigma M_D = 0$



$$-M_D - (500)(1-a) = 0$$

$$M_D = -500(1-a)$$

(a) Equate $\quad -M_D = M_C \qquad 500(1-a) = 500 - \frac{375}{a}$

$$a = 0.86603 \text{ m} \qquad a = 866 \text{ mm}$$

$A = 133.98$ kN $\qquad M_C = 66.985$ kN·m $\qquad M_D = -66.985$ kN·m

For rectangular cross section $\quad S = \frac{1}{6}bh^3 = \frac{1}{6}(12)(18)^3 = 11.664 \times 10^3 \text{ mm}^3$

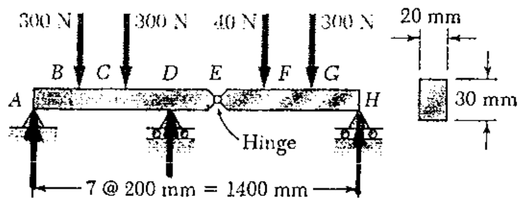
$$= 11.664 \times 10^{-6} \text{ m}^3$$

(b) Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{66.985 \times 10^3}{11.664 \times 10^{-6}} = 5.74 \times 10^6 \text{ Pa}$

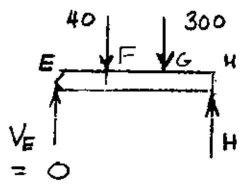
$$= 5.74 \text{ MPa}$$

Problem 5.30

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Free body EFGH



Note that $M_E = 0$ due to hinge.

$\circlearrowleft \Sigma M_E = 0$

$0.6\,H - (0.2)(40) - (0.4)(300) = 0$

$H = 213.33$ N

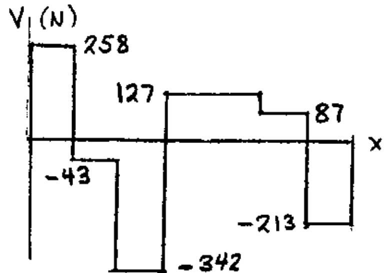
$+\uparrow \Sigma F_y = 0 \qquad V_E - 40 - 300 + 213.33 = 0$

$V_E = 126.67$ N

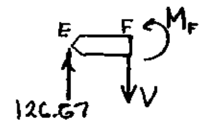
Shear: E to F $\quad V = 126.67$ N·m

F to G $\quad V = 86.67$ N·m

G to H $\quad V = -213.33$ N·m



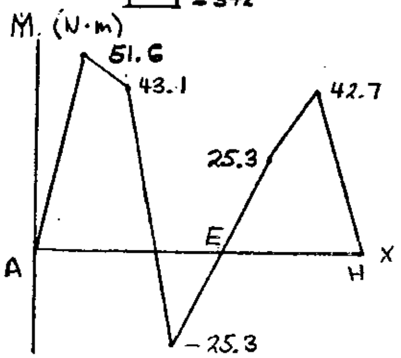
Bending moment at F



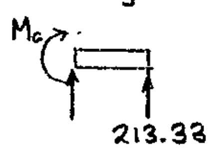
$\circlearrowleft \Sigma M_F = 0$

$M_F - (0.2)(126.67) = 0$

$M_F = 25.33$ N·m



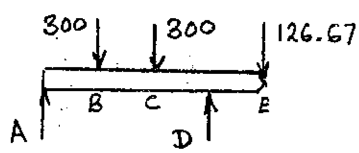
Bending moment at G



$\circlearrowleft \Sigma M_G = 0$

$-M_G + (0.2)(213.33) = 0$

$M_G = 42.67$ N·m



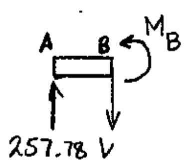
Free body ABCDE

$+\circlearrowleft \Sigma M_B = 0 \qquad -0.6\,A + (0.4)(300) + (0.2)(300) - (0.2)(126.67) = 0$

$A = 257.78$ N

$+\circlearrowleft \Sigma M_A = 0 \qquad -(0.2)(300) - (0.4)(300) - (0.8)(126.67) + 0.6\,D = 0$

$D = 468.89$ N

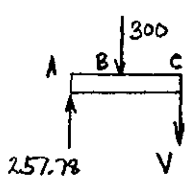


Bending moment at B

$\circlearrowleft \Sigma M_B = 0$

$-(0.2)(257.78) + M_B = 0$

$M_B = 51.56$ N·m

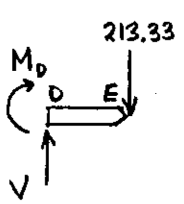


Bending moment at C

$+\circlearrowleft \Sigma M_C = 0$

$-(0.4)(257.78) + (0.2)(300) + M_C = 0$

$M_C = 43.11$ N·m



Bending moment at D

$+\circlearrowleft \Sigma M_D = 0$

$-M_D - (0.2)(213.33) = 0$

$M_D = -25.33$ N·m

max $|M| = 51.56$ N·m

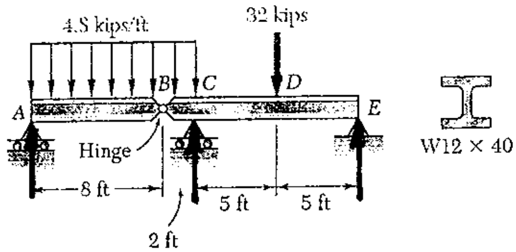
$S = \frac{1}{6} b h^2 = \frac{1}{6}(20)(30)^2 = 3 \times 10^3 \text{ mm}^3 = 3 \times 10^{-6} \text{ m}^3$

Normal stress

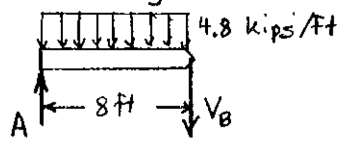
$\sigma = \dfrac{51.56}{3 \times 10^{-6}} = 17.19 \times 10^6$ Pa $= 17.19$ MPa ◄

Problem 5.31

5.30 and 5.31 Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.



Free body AB



$\circlearrowleft \Sigma M_B = 0$

$(4.8)(8)(4) - 8A = 0$

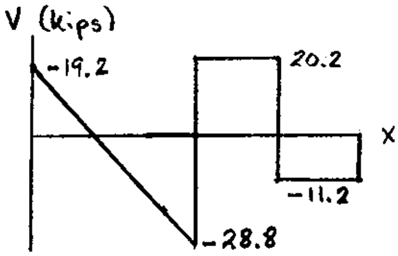
$A = 19.2$ kips

$\circlearrowleft M_A = 0$

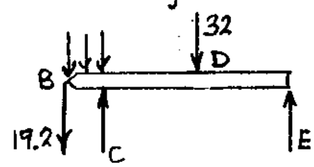
$-(4.8)(8)(4) - 8V_B = 0$

$V_B = -19.2$ kips

V (kips)



Free body BCDE



$+\circlearrowleft \Sigma M_E = 0$

$(19.2)(12) + (4.8)(2)(11)$

$- 10C + (32)(5) = 0$

$C = 49.2$ Kips

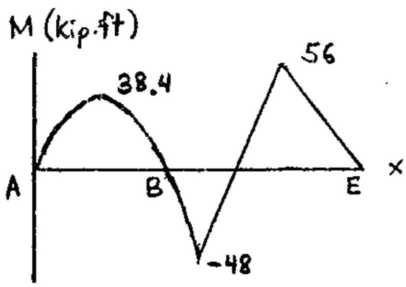
$+\circlearrowleft M_c = 0$

$(19.2)(2) + (4.8)(2)(1)$

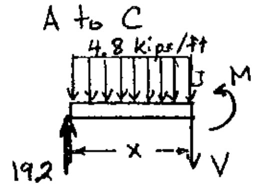
$-(32)(5) + 10E = 0$

$E = 11.2$ Kips

M (kip·ft)



A to C



$0 < x < 10$ ft.

$+\uparrow \Sigma F_y = 0$

$19.2 - 4.8x - V = 0$

$V = 19.2 - 4.8x$ kips.

$\circlearrowleft M_J = 0$

$-19.2x + (4.8x)\frac{x}{2} + M = 0$

$M = 19.2x - 2.4x^2$ kip·ft

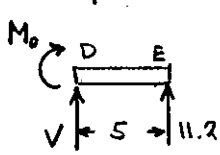
At C^- $x = 10$ $V = 19.2 - (4.8)(10) = -28.8$ Kips

At C $x = 10$ $M_c = (19.2)(10) - (2.4)(10)^2 = -48$ kip·ft

C to D $V = 19.2 - (4.8)(10) + 49.6 = 20.8$ Kips.

D to E $V = -11.2$ kips

Bending moment at D



$\circlearrowleft \Sigma M_D = 0$

$-M_D + (11.2)(5) = 0$

$M_D = 56$ kip·ft

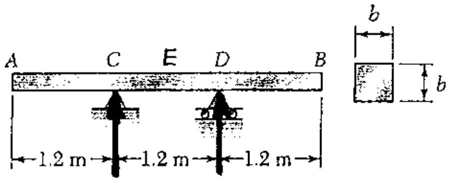
max $|M| = 56$ kip·ft $= 672$ kip·in

For W 12× 40 rolled steel section $S = 51.9$ in^3

Normal stress $\sigma = \frac{|M|}{S} = \frac{672}{51.9} = 12.95$ ksi ◄

Problem 5.32

5.32 A solid steel bar has a square cross section of side b and is supported as shown. Knowing that for steel $\rho = 7860\ \text{kg/m}^3$, determine the dimension b for which the maximum normal stress due to bending is (*a*) 10 MPa, (*b*) 50 MPa.

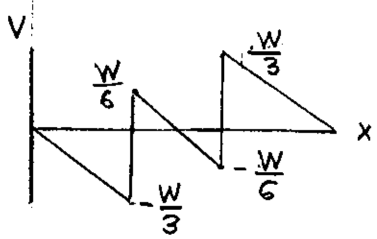


Weight density $\gamma = \rho g$

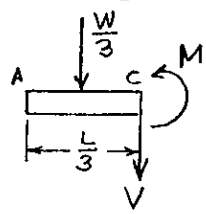
Let L = total length of beam

$$W = \mathcal{V}\gamma = AL\rho g = b^2 L \rho g$$

Reactions at C and D $\quad C = D = \frac{W}{2}$



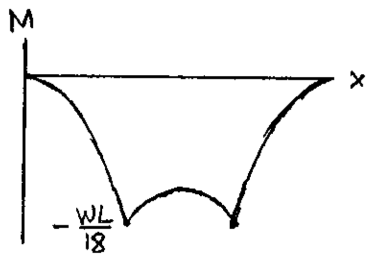
Bending moment at C



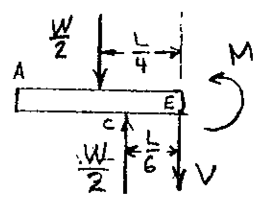
$$\circlearrowleft \Sigma M_C = 0$$

$$\left(\frac{L}{6}\right)\left(\frac{W}{3}\right) + M = 0$$

$$M = -\frac{WL}{18}$$



Bending moment at center of beam



$$\circlearrowleft \Sigma M_E = 0$$

$$\left(\frac{L}{4}\right)\left(\frac{W}{2}\right) - \left(\frac{L}{6}\right)\left(\frac{W}{2}\right) + M = 0$$

$$M = -\frac{WL}{24}$$

$$\max|M| = \frac{WL}{18} = \frac{b^2 L^2 \rho g}{18}$$

For a square section $\quad S = \frac{1}{6} b^3$

Normal stress $\quad \sigma = \frac{|M|}{S} = \frac{b^2 L^2 \rho g / 18}{b^3/6} = \frac{L^2 \rho g}{3b}$

Solve for b $\quad b = \frac{L^2 \rho g}{3\sigma}$

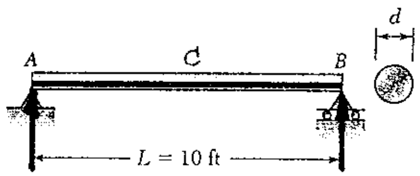
Data: $\quad L = 3.6\ \text{m} \qquad \rho = 7860\ \text{kg/m}^3 \qquad g = 9.81\ \text{m/s}^2$

(a) $\sigma = 10 \times 10^6\ \text{Pa}$ $\qquad$ (b) $\sigma = 50 \times 10^6\ \text{Pa}$

(a) $\quad b = \frac{(3.6)^2(7860)(9.81)}{(3)(10\times10^6)} = 33.3 \times 10^{-3}\ \text{m} = 33.3\ \text{mm}$ ◀

(b) $\quad b = \frac{(3.6)^2(7860)(9.81)}{(3)(50\times10^6)} = 6.66 \times 10^{-3}\ \text{m} = 6.66\ \text{mm}$ ◀

Problem 5.33



5.33 A solid steel rod of diameter d is supported as shown. Knowing that for steel $\gamma = 490\ \text{lb/ft}^3$, determine the smallest diameter d that can be used if the normal stress due to bending is not to exceed 4 ksi.

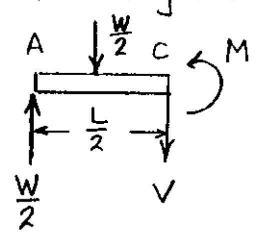
Let W = total weight

$$W = \mathcal{V}\gamma = AL\gamma = \frac{\pi}{4} d^2 L \gamma$$

Reaction at A

$$A = \frac{1}{2} W$$

Bending moment at center of beam



$$+\circlearrowleft \Sigma M_C = 0$$

$$-\left(\frac{W}{2}\right)\left(\frac{L}{2}\right) + \left(\frac{W}{2}\right)\left(\frac{L}{4}\right) + M = 0$$

$$M = \frac{WL}{8} = \frac{\pi}{32} d^2 L^2 \gamma$$

For circular cross section $\quad (c = \frac{1}{2} d)$

$$I = \frac{\pi}{4} c^4, \qquad S = \frac{I}{c} = \frac{\pi}{4} c^3 = \frac{\pi}{32} d^3$$

Normal stress

$$\sigma = \frac{M}{S} = \frac{\frac{\pi}{32} d^2 L^2 \gamma}{\frac{\pi}{32} d^3} = \frac{L^2 \gamma}{d}$$

Solving for d $\qquad d = \dfrac{L^2 \gamma}{\sigma}$

Data: $\quad L = 10\ \text{ft} = (12)(10) = 120\ \text{in}$

$$\gamma = 490\ \text{lb/ft}^3 = \frac{490}{12^3} = 0.28356\ \text{lb/in}^3$$

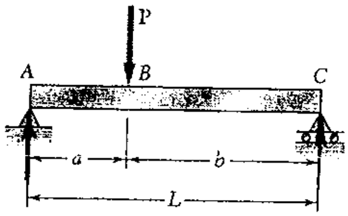
$$\sigma = 4\ \text{ksi} = 4000\ \text{lb/in}^2$$

$$d = \frac{(120)^2 (0.28356)}{4000} = 1.021\ \text{in.}$$

Problem 5.34

5.34 Using the method of Sec. 5.3, solve Prob. 5.1*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$\circlearrowleft \Sigma M_C = 0 \qquad LA - bP = 0 \qquad A = \frac{Pb}{L}$

$\circlearrowleft \Sigma M_A = 0 \qquad LC - aP = 0 \qquad C = \frac{Pa}{L}$

At A^+ $\quad V = A = \frac{Pb}{L} \qquad M = 0$

A to B^- $\quad 0 < x < a$

$$w = 0 \qquad \int_0^x w\,dx = 0$$

$$V_B - V_A = 0 \qquad V_B = \frac{Pb}{L}$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{Pb}{L}\,dx = \frac{Pba}{L}$$

$$M_B = \frac{Pba}{L}$$

At B^+ $\quad V = A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$

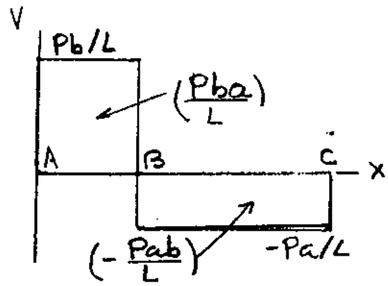
B^+ to C $\quad a < x < L$

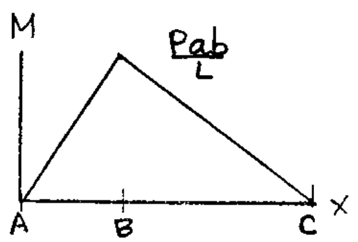
$$w = 0 \qquad \int_a^x w\,dx = 0$$

$$V_C - V_B = 0 \qquad V = -\frac{Pa}{L}$$

$$M_C - M_B = \int_a^L V\,dx = -\frac{Pa}{L}(L-a) = -\frac{Pab}{L}$$

$$M_C = M_B - \frac{Pab}{L} = \frac{Pba}{L} - \frac{Pab}{L} = 0$$

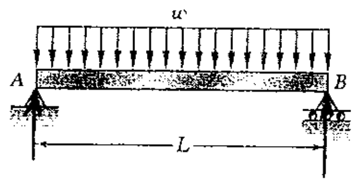




Problem 5.35

5.35 Using the method of Sec. 5.3, solve Prob. 5.2*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$\circlearrowleft \Sigma M_B = 0 \qquad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

$$\circlearrowleft \Sigma M_A = 0 \qquad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = V_A - wx = A - wx = \frac{wL}{2} - wx$$

$$\frac{dM}{dx} = V$$

$$M - M_A = \int_0^x V\,dx = \int_0^x \left(\frac{wL}{2} - wx\right)dx$$

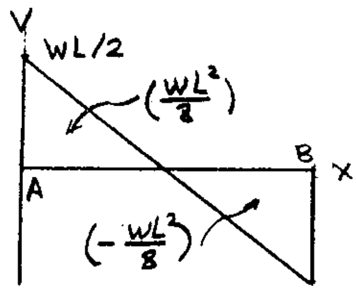
$$= \frac{wLx}{2} - \frac{wx^2}{2}$$

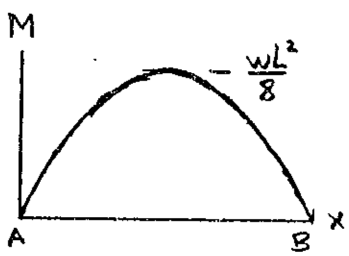
$$M = M_A + \frac{wLx}{2} - \frac{wx^2}{2} = \frac{w}{2}(Lx - x^2)$$

Maximum M occurs at $x = \frac{L}{2}$ where

$$V = \frac{dM}{dx} = 0$$

$$\text{Max } M = \frac{wL^2}{8}$$

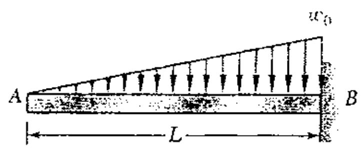




Problem 5.36

5.36 Using the method of Sec. 5.3, solve Prob. 5.3*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$w = w_0 \frac{x}{L}$$

$$V_A = 0, \qquad M_A = 0$$

$$\frac{dV}{dx} = -w = -\frac{w_0 x}{L}$$

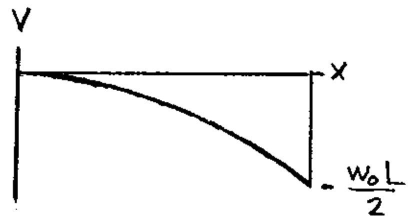
$$V - V_A = -\int_0^x \frac{w_0 x}{L} = -\frac{w_0 x^2}{2L}$$

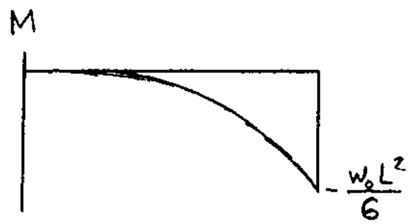
$$V = -\frac{w_0 x^2}{2L}$$

$$\frac{dM}{dx} = V = -\frac{w_0 x^2}{2L}$$

$$M - M_A = \int_0^x V\,dx = -\int_0^x \frac{w_0 x^2}{2L}\,dx$$

$$= -\frac{w_0 x^3}{6L}$$

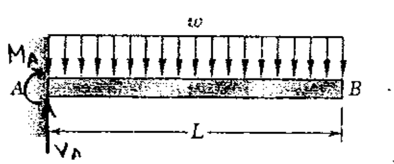




Problem 5.37

5.37 Using the method of Sec. 5.3, solve Prob. 5.4*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



$$+\uparrow \Sigma F_y = 0 \qquad V_A - wL = 0 \qquad V_A = wL$$

$$\circlearrowleft \Sigma M_A = 0 \qquad -M - (wL)(\tfrac{L}{2}) = 0 \qquad M_A = -\frac{wL^2}{2}$$

$$\frac{dV}{dx} = -w$$

$$V - V_A = -\int_0^x w\,dx = -wx$$

$$V = wL - wx$$

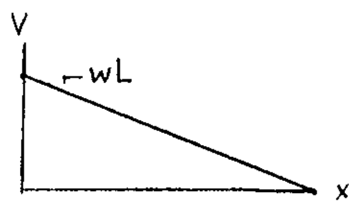
$$\frac{dM}{dx} = V = wL - wx$$

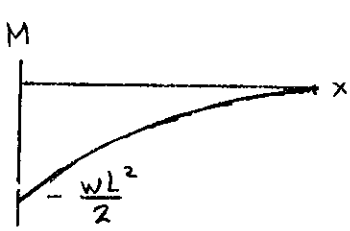
$$M - M_A = \int_0^x (wL - wx)\,dx = wLx - \frac{wx^2}{2}$$

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

$$\max |V| = wL$$

$$\max |M| = \frac{wL^2}{2}$$

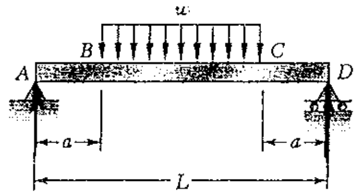




Problem 5.38

5.38 Using the method of Sec. 5.3, solve Prob. 5.5*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.



Reactions $\quad A = D = \frac{1}{2}w(L-2a)$

At A $\quad V_A = A = \frac{1}{2}w(L-2a), \quad M_A = 0$

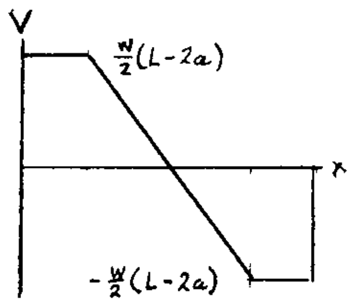
A to B $\quad 0 < x < a \qquad w = 0$

$$V_B - V_A = -\int_0^a w\,dx = 0$$

$$V_B = V_A = \frac{1}{2}w(L-2a)$$

$$M_B - M_A = \int_0^a V\,dx = \int_0^a \frac{1}{2}w(L-2a)\,dx$$

$$M_B = \frac{1}{2}w(L-2a)a$$



B to C $\quad a < x < L-a \qquad w = w$

$$V - V_B = -\int_a^x w\,dx = -w(x-a)$$

$$V = \frac{1}{2}w(L-2a) - w(x-a) = \frac{1}{2}w(L-2x)$$

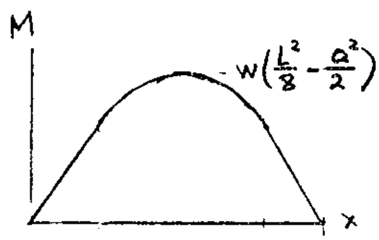
$$\frac{dM}{dx} = V = \frac{1}{2}w(L-2x)$$

$$M - M_B = \int_a^x V\,dx = \frac{1}{2}w(Lx - x^2)\Big|_a^x$$

$$= \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$M = \frac{1}{2}w(L-2a)a + \frac{1}{2}w(Lx - x^2 - La + a^2)$$

$$= \frac{1}{2}w(Lx - x^2 - a^2)$$



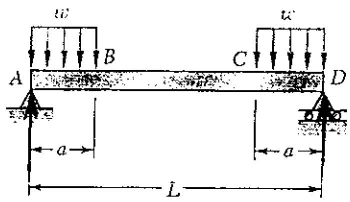
At C $\quad x = L-a \qquad V_c = -\frac{1}{2}w(L-2a) \qquad M_c = \frac{1}{2}(L-2a)a$

C to D $\quad V = V_c = -\frac{1}{2}w(L-2a)$

$$M_D = 0$$

At $x = \frac{L}{2} \qquad M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right)$ ◄

Problem 5.39



5.39 Using the method of Sec. 5.3, solve Prob. 5.6*a*.

5.1 through 5.6 Draw the shear and bending-moment diagrams for the beam and loading shown.

Reactions $A = D = wa$

A to B $0 < x < a$ $w = w$

$V_A = A = wa,$ $M_A = 0$

$V - V_A = -\int_0^x w\,dx = -wx$

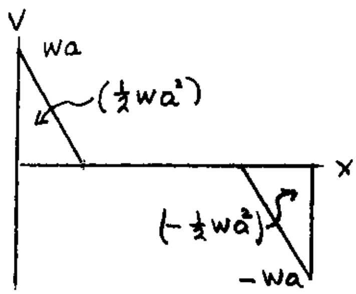
$V = w(a-x)$ $V_B = 0$

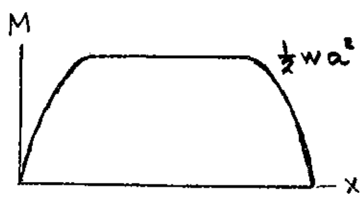
$\frac{dM}{dx} = V = wa - wx$

$M - M_A = \int_0^x V\,dx = \int_0^x (wa - wx)\,dx$

$= wax - \frac{1}{2}wx^2$

$M_B = \frac{1}{2}wa^2$ at $x = a.$





B to C $a < x < L-a$ $V = 0$

$\frac{dM}{dx} = V = 0$

$M - M_B = \int_a^x V\,dx = 0$

$M = M_B = \frac{1}{2}wa^2$

C to D $V - V_C = -\int_{L-a}^{x} w\,dx = -w[x - (L-a)]$

$V = -w[x - (L-a)]$

$M - M_C = \int_{L-a}^{x} V\,dx = \int_{L-a}^{x} -w[x - (L-a)]\,dx$

$= -w\left[\frac{x^2}{2} - (L-a)x\right]\Big|_{L-a}^{x}$

$= -w\left[\frac{x^2}{2} - (L-a)x - \frac{(L-a)^2}{2} + (L-a)^2\right]$

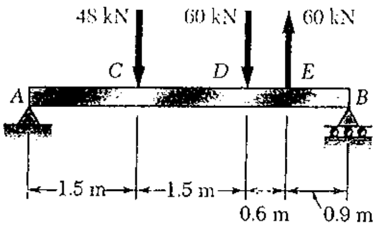
$= -w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$

$M = \frac{1}{2}wa^2 - w\left[\frac{x^2}{2} - (L-a)x + \frac{(L-a)^2}{2}\right]$

Problem 5.40

5.40 Using the method of Sec. 5.3, solve Prob. 5.7.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

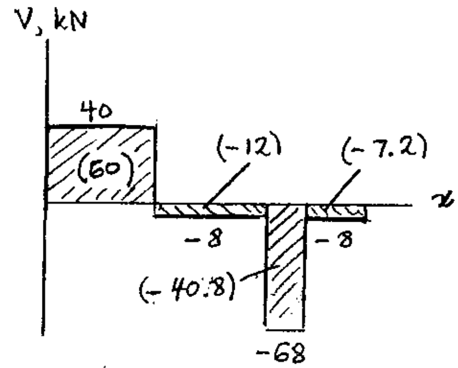


Reactions.

$\circlearrowleft + \Sigma M_B = 0$

$-1.5 R_A + (3.0)(48) + (1.5)(60) - (0.9)(60) = 0$

$R_A = 40 \text{ kN}$

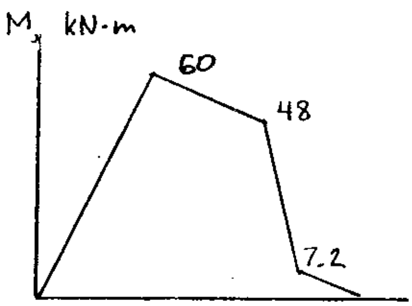


Shear

A to C^-	$V = 40 \text{ kN}$
C^+ to D^-	$V = 40 - 48 = -8 \text{ kN}$
D^+ to E^-	$V = -8 - 60 = -68 \text{ kN}$
E^+ to B	$V = -68 + 60 = -8 \text{ kN}$

Areas of Shear Diagram.

A to C	$A_{AC} = (1.5)(40) = 60 \text{ kN}\cdot\text{m}$
C to D	$A_{CD} = (1.5)(-8) = -12 \text{ kN}\cdot\text{m}$
D to E	$A_{DE} = (0.6)(-68) = -40.8 \text{ kN}\cdot\text{m}$
E to B	$A_{EB} = (0.9)(-8) = -7.2 \text{ kN}\cdot\text{m}$



Bending moments.

$M_A = 0$

$M_C = 0 + 60 = 60 \text{ kN}\cdot\text{m}$

$M_D = 60 - 12 = 48 \text{ kN}\cdot\text{m}$

$M_E = 48 - 40.8 = 7.2 \text{ kN}\cdot\text{m}$

$M_B = 7.2 - 7.2 = 0$

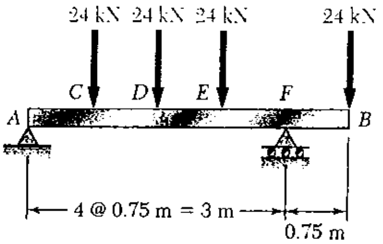
From the diagrams (a) $|V|_{max} = 68.0 \text{ kN}\cdot\text{m}$ ◄

(b) $|M|_{max} = 60.0 \text{ kN}\cdot\text{m}$ ◄

Problem 5.41

5.41 Using the method of Sec. 5.3, solve Prob. 5.8.

5.7 and 5.8 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reactions at A and F.

$+\circlearrowleft \Sigma M_F = 0$

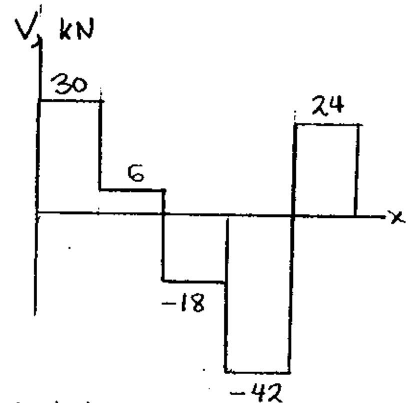
$-3R_A + (2.25)(24) + (1.50)(24) + (0.75)(24)$
$\quad -(0.75)(24) = 0$

$R_A = 30 \text{ kN} \uparrow$

$+\circlearrowleft \Sigma M_A = 0$

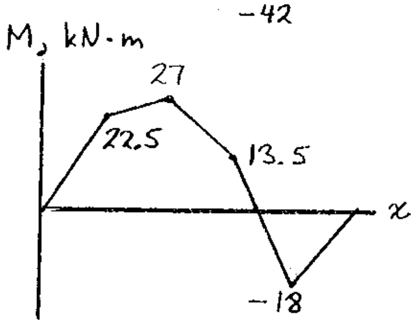
$-(0.75)(24) - (1.50)(24) - (2.25)(24) + 3R_F$
$\quad -(3.75)(24) = 0$

$R_F = 66 \text{ kN} \uparrow$



Shear diagram.

A to C	$V = 30 \text{ kN}$
C to D	$V = 30 - 24 = 6 \text{ kN}$
D to E	$V = 6 - 24 = -18 \text{ kN}$
E to F	$V = -18 - 24 = -42 \text{ kN}$
F to B	$V = -42 + 66 = 24 \text{ kN}$



Areas of shear diagram

A to C	$A_{AC} = (0.75)(30) = 22.5 \text{ kN·m}$
C to D	$A_{CD} = (0.75)(6) = 4.5 \text{ kN·m}$
D to E	$A_{DE} = (0.75)(-18) = -13.5 \text{ kN·m}$
E to F	$A_{EF} = (0.75)(-42) = -31.5 \text{ kN·m}$
F to B	$A_{FB} = (0.75)(24) = 18 \text{ kN·m}$

Bending moments.

$M_A = 0$

$M_C = 0 + 22.5 = 22.5 \text{ kN·m}$

$M_D = 22.5 + 4.5 = 27 \text{ kN·m}$

$M_E = 27 - 13.5 = 13.5 \text{ kN·m}$

$M_F = 13.5 - 31.5 = -18 \text{ kN·m}$

$M_B = -18 + 18 = 0$

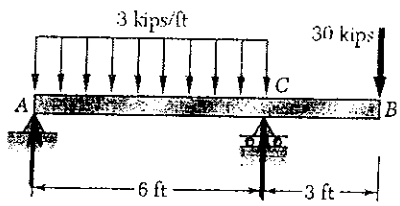
(a) $|V|_{max} = 42.0 \text{ kN}$ ◄

(b) $|M|_{max} = 27.0 \text{ kN·m}$ ◄

Problem 5.42

5.42 Using the method of Sec. 5.3, solve Prob. 5.9.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

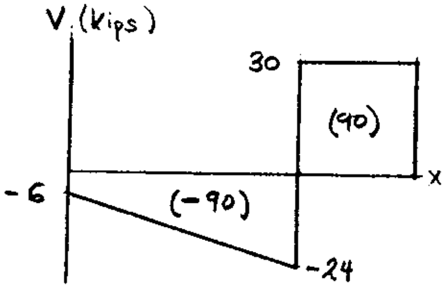


$+\circlearrowleft \Sigma M_C = 0 \quad -6A + (3)(18) - (3)(30) = 0$

$A = -6 \text{ kips} \quad \text{i.e.} \quad 6 \text{ kips} \downarrow$

$+\circlearrowleft \Sigma M_A = 0 \quad 6C - (3)(18) - (9)(30) = 0$

$C = 54 \text{ kips} \uparrow$



Shear

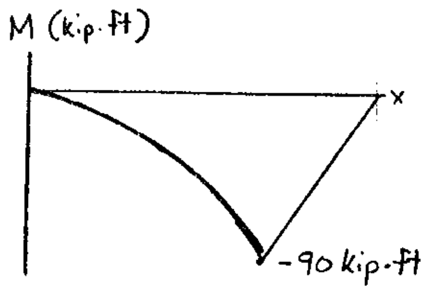
$V_A = -6 \text{ kips}$

A to C $\quad 0 < x < 6 \text{ ft.} \quad w = -3 \text{ kips/ft}$

$V_B - V_A = -\int_0^6 w\,dx = -\int_0^6 3\,dx = -18 \text{ kips}$

$V_B = -6 - 18 = -24 \text{ kips}$

C to B $\quad V = -24 + 54 = 30 \text{ kips.}$



Areas under shear diagram

A to C $\quad \int V dx = (\frac{1}{2})(-6-24)(6)$

$= -90 \text{ kip ft.}$

C to B $\quad \int V\,dx = (3)(30) = 90 \text{ kip·ft}$

Bending moments

$M_A = 0$

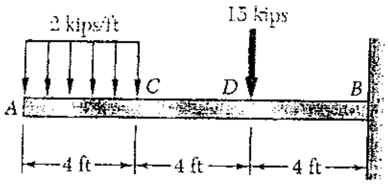
$M_C = M_A + \int V dx = 0 - 90 = -90 \text{ kip·ft}$

$M_B = M_C + \int V dx = -90 + 90 = 0$

Maximum $|V| = 30$ kips ◄

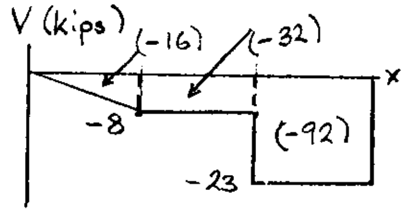
Maximum $|M| = 90$ kip·ft ◄

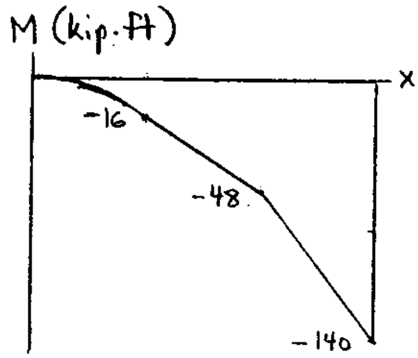
Problem 5.43



5.43 Using the method of Sec. 5.3, solve Prob. 5.10.

5.9 and 5.10 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.





Shear

$V_A = 0$

$V_B = V_A - \int_A^B w\,dx = 0 - (4)(2) = -8$ kips.

C to D $\quad V = -8$ kips

D to B $\quad V = -8 - 15 = -23$ kips

Areas under shear diagram

A to C $\quad \int V\,dx = (\frac{1}{2})(4)(-8) = -16$ kip·ft

C to D $\quad \int V\,dx = (4)(-8) = -32$ kip ft.

D to B $\quad \int V\,dx = (4)(-23) = -92$ kip·ft

Bending moments

$M_A = 0$

$M_C = M_A + \int V\,dx = 0 - 16 = -16$ kip ft.

$M_D = M_C + \int V\,dx = -16 - 32 = -48$ kip·ft

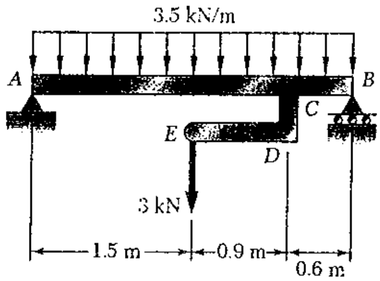
$M_B = M_D + \int V\,dx = -48 - 92 = -140$ kip·ft

Maximum $|V| = 23$ kips ◄

Maximum $|M| = 140$ kip·ft ◄

Problem 5.44

5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.



Reaction at A.

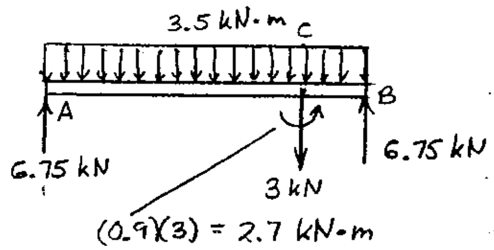
$+\circlearrowleft \sum M_B = 0$

$-3.0\,A + (1.5)(3.0)(3.5) + (1.5)(3) = 0$

$A = 6.75 \text{ kN}$

Reaction at B. $\quad B = 6.75 \text{ kN}$

Beam ACB and loading. See sketch.



Areas of load diagram.

A to C $\quad (2.4)(3.5) = 8.4 \text{ kN}$

C to B $\quad (0.6)(3.5) = 2.1 \text{ kN}$

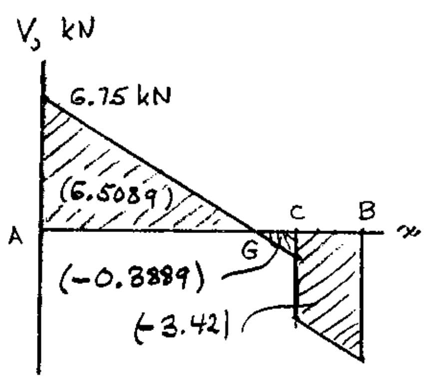
Shear diagram.

$V_A = 6.75 \text{ kN}$

$V_C^- = 6.75 - 8.4 = -1.65 \text{ kN}$

$V_C^+ = -1.65 - 3 = -4.65 \text{ kN}$

$V_B = -4.65 - 2.1 = -6.75 \text{ kN}$



Over A to C $\quad V = 6.75 - 3.5x$

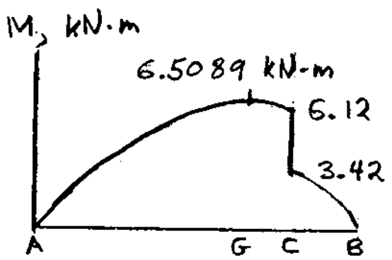
At G $\quad V = 6.75 - 3.5x_G = 0 \quad x_G = 1.9286 \text{ m}$

Areas of shear diagram.

A to G $\quad \frac{1}{2}(1.9286)(6.75) = 6.5089 \text{ kN}\cdot\text{m}$

G to C $\quad \frac{1}{2}(0.4714)(-1.65) = -0.3889 \text{ kN}\cdot\text{m}$

C to B $\quad \frac{1}{2}(0.6)(-4.65 - 6.75) = -3.42 \text{ kN}\cdot\text{m}$



Bending moments.

$M_A = 0$

$M_G = 0 + 6.5089 = 6.5089 \text{ kN}\cdot\text{m}$

$M_C^- = 6.5089 - 0.3889 = 6.12 \text{ kN}\cdot\text{m}$

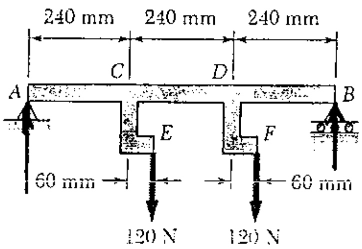
$M_C^+ = 6.12 - 2.7 = 3.42 \text{ kN}\cdot\text{m}$

$M_B = 3.42 - 3.42 = 0$

(a) $|V|_{max} = 6.75 \text{ kN}$ ◄

(b) $|M|_{max} = 6.51 \text{ kN}\cdot\text{m}$ ◄

Problem 5.45



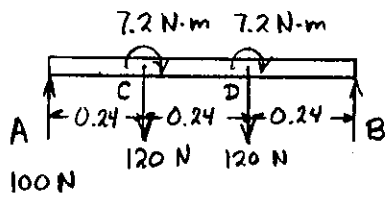
5.44 and 5.45 Draw the shear and bending-moment diagrams for the beam and loading shown, and determine the maximum absolute value (*a*) of the shear, (*b*) of the bending moment.

$+\circlearrowleft \Sigma M_B = 0 \qquad -0.72A + (0.48)(120) + (0.24)(120) - 7.2 - 7.2 = 0$

$A = 100 \text{ N}$

$+\circlearrowleft \Sigma M_A = 0 \qquad -(0.24)(120) - (0.48)(120) - 7.2 - 7.2 + 0.72B = 0$

$B = 140 \text{ N}$



Shear

A to C $\quad V = 100 \text{ N}$

C to D $\quad V = 100 - 120 = -20 \text{ N}$

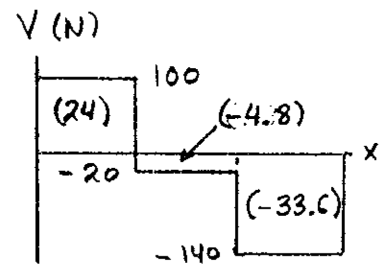
D to B $\quad V = -20 - 120 = -140 \text{ N}$

Areas under shear diagram

A to C $\quad \int V\,dx = (0.24)(100) = 24 \text{ N·m}$

C to D $\quad \int V\,dx = (0.24)(-20) = -4.8 \text{ N·m}$

D to B $\quad \int V\,dx = (0.24)(-140) = -33.6 \text{ N·m}$



Bending moments

$M_A = 0$

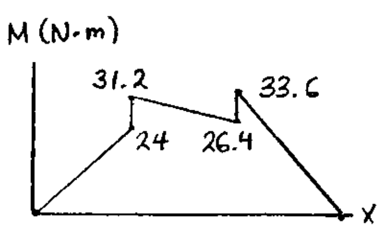
$M_C^- = 0 + 24 = 24 \text{ N·m}$

$M_C^+ = 24 + 7.2 = 31.2 \text{ N·m}$

$M_D^- = 31.2 - 4.8 = 26.4 \text{ N·m}$

$M_D^+ = 26.4 + 7.2 = 33.6 \text{ N·m}$

$M_B = 33.6 - 33.6 = 0$



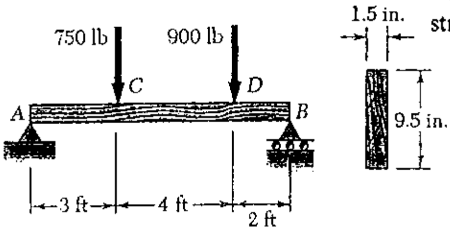
Maximum $|V| = 140 \text{ N}$ ◄

Maximum $|M| = 33.6 \text{ N·m}$ ◄

Problem 5.46

5.46 Using the method of Sec. 5.3, solve Prob. 5.15.

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.

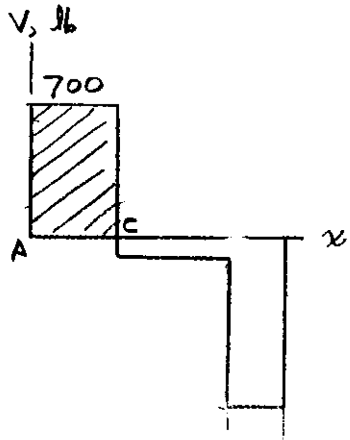


Reaction at A.

$$+\circlearrowleft \Sigma M_B = 0$$

$$-9A + (6)(750) + (2)(900) = 0$$

$$A = 700 \text{ lb.}$$



Shear curve from A to C: $V = 700$ lb.

Area of shear curve from A to C

$$A_{AC} = (3)(700) = 2100 \text{ lb}\cdot\text{ft}$$

Bending moments. $M_A = 0$

$$M_C = M_A + A_{AC} = 0 + 2100 = 2100 \text{ lb}\cdot\text{ft}$$

$$= 25.2\times10^3 \text{ lb}\cdot\text{in} = 25.2 \text{ kip}\cdot\text{in.}$$

For the cross section $I = \frac{1}{12}(1.5)(9.5)^3 = 107.172 \text{ in}^4$

$$c = 4.75 \text{ in.}$$

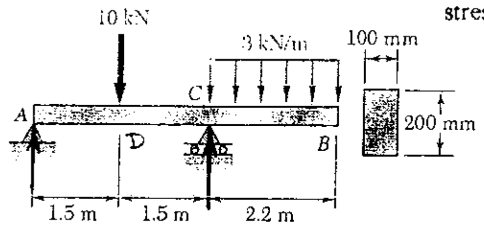
Maximum normal stress due to bending.

$$\sigma = \frac{Mc}{I} = \frac{(25.2)(4.75)}{107.172} \qquad \sigma = 1.117 \text{ ksi}$$

Problem 5.47

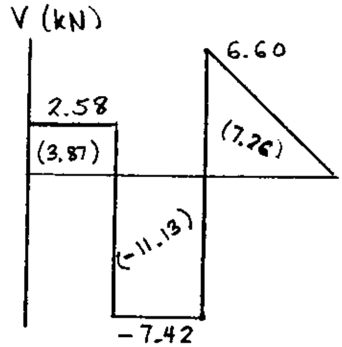
5.47 Using the method of Sec. 5.3, solve Prob. 5.16.

5.15 and *5.16* For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at C.



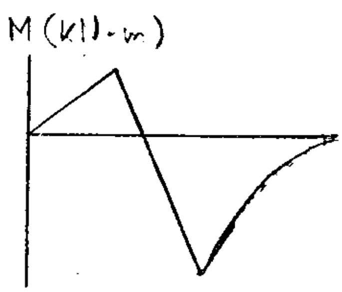
$$+\circlearrowleft \sum M_C = 0$$
$$-3A + (1.5)(10) - (1.1)(2.2)(3) = 0$$
$$A = 2.58 \text{ kN}$$

$$+\circlearrowleft \sum M_A = 0$$
$$-(1.5)(10) + 3C - (4.1)(2.2)(3) = 0$$
$$C = 14.02 \text{ kN}$$



Shear

A to D^-	$V = 2.58$ kN
D^+ to C^-	$V = 2.58 - 10 = -7.42$ kN
C^+	$V = -7.42 + 14.02 = 6.60$ kN
B	$V = 6.60 - (2.2)(3) = 0$



Areas under shear diagram

A to D	$\int V dx = (1.5)(2.58) = 3.87$ kN·m
D to C	$\int V dx = (1.5)(-7.42) = -11.13$ kN·m
C to B	$\int V dx = (\frac{1}{2})(2.2)(6.60) = 7.26$ kN·m

Bending moments

$$M_A = 0$$
$$M_D = 0 + 3.87 = 3.87 \text{ kN·m}$$
$$M_C = 3.87 - 11.13 = -7.26 \text{ kN·m}$$
$$M_B = 7.26 - 7.26 = 0$$

$$|M_C| = 7.26 \text{ kN·m} = 7.26 \times 10^3 \text{ N·m}$$

For rectangular cross section $S = \frac{1}{6}bh^2 = (\frac{1}{6})(100)(200)^2$

$$= 666.67 \times 10^3 \text{ mm}^3 = 666.67 \times 10^{-6} \text{ m}^3$$

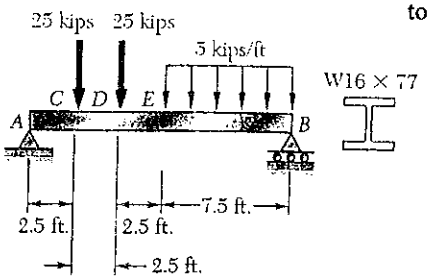
Normal stress $\sigma = \dfrac{|M_C|}{S} = \dfrac{7.26 \times 10^3}{666.67 \times 10^{-6}} = 10.89 \times 10^6$ Pa

$$= 10.89 \text{ MPa}$$

Problem 5.48

5.48 Using the method of Sec. 5.3, solve Prob. 5.17.

5.17 For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.



$$+\circlearrowleft \Sigma M_B = 0$$

$$-15A + (12.5)(25) + (10)(25) + (3.75)(7.5)(5) = 0$$

$$A = 46.875 \text{ kips}$$

Shear A to C $\quad V = 46.875$ kips

Area under shear curve A to C $\quad \int V\,dx = (2.5)(46.875)$
$= 117.1875$ kip·ft

$M_A = 0$

$M_C = 0 + 117.1875 = 117.1875 \text{ kip·ft} = 1406.25 \text{ kip·in}$

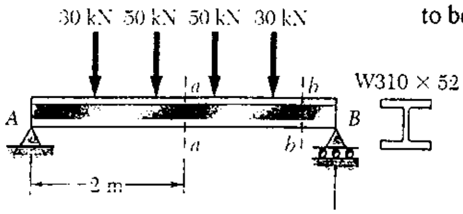
For W 16×77 rolled steel section $\quad S = 134 \text{ in}^3$

Normal stress $\quad \sigma = \dfrac{M}{S} = \dfrac{1406.25}{134} = 10.49$ ksi ◄

Problem 5.49

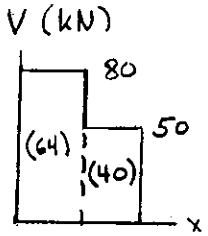
5.49 Using the method of Sec. 5.3, solve Prob. 5.18.

5.18 For the beam and loading shown, determine the maximum normal stress due to bending on section *a-a*.



Reactions: By symmetry $A = B$

$+\uparrow \Sigma F_y = 0 \qquad A = B = 80 \text{ kN}$



Shear

A to C $\quad V = 80$ kN

C to D $\quad V = 80 - 30 = 50$ kN

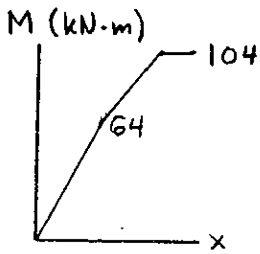
D to E $\quad V = 50 - 50 = 0$

Areas under shear diagram

A to C $\quad \int V dx = (80)(0.8) = 64$ kN·m

C to D $\quad \int V dx = (50)(0.8) = 40$ kN·m

D to E $\quad \int V dx = 0$



Bending moments

$M_A = 0$

$M_C = 0 + 64 = 64$ kN·m

$M_D = 64 + 40 = 104$ kN·m

$M_E = 104 + 0 = 104$ kN·m

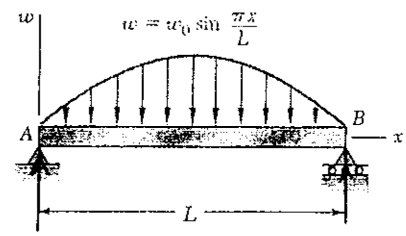
$M_E = 104$ kN·m $= 104 \times 10^3$ N·m

For W 310 × 52 $\quad S = 748 \times 10^3 \text{ mm}^3 = 748 \times 10^{-6} \text{ m}^3$

Normal stress $\quad \sigma = \dfrac{|M|}{S} = \dfrac{104 \times 10^3}{748 \times 10^{-6}} = 139.0 \times 10^6 \text{ Pa} = 139.0 \text{ MPa}$ ◀

Problem 5.50

5.50 and *5.51* Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -W_o \sin \frac{\pi x}{L}$$

$$V = \frac{W_o L}{\pi} \cos \frac{\pi x}{L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{W_o L^2}{\pi^2} \sin \frac{\pi x}{L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = 0 + C_1 L + 0 \qquad C_1 = 0$

$$V = \frac{W_o L}{\pi} \cos \frac{\pi x}{L}$$

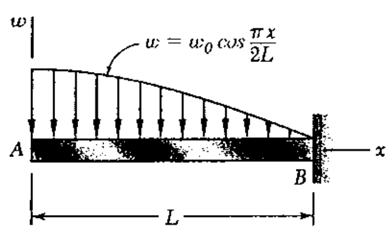
$$M = \frac{W_o L^2}{\pi^2} \sin \frac{\pi x}{L}$$

$$\frac{dM}{dx} = V = 0 \quad \text{at} \quad x = \frac{L}{2}$$

$$M_{max} = \frac{W_o L^2}{\pi^2} \sin \frac{\pi}{2} = \frac{W_o L^2}{\pi^2}$$

Problem 5.51

5.50 and *5.51* Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$\frac{dV}{dx} = -w = -W_o \cos \frac{\pi x}{2L}$$

$$V = -\frac{2LW_o}{\pi} \sin \frac{\pi x}{2L} + C_1 = \frac{dM}{dx}$$

$$M = \frac{4L^2 W_o}{\pi^2} \cos \frac{\pi x}{2L} + C_1 x + C_2$$

$V = 0$ at $x = 0$ $\qquad$ Hence $C_1 = 0$

$M = 0$ at $x = 0$. $\qquad$ Hence $C_2 = -\frac{4L^2 W_o}{\pi^2}$

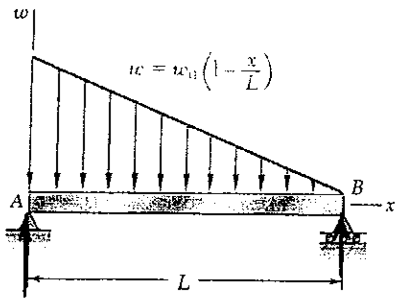
(a) $\qquad V = -(2LW_o/\pi)\sin(\pi x/2L)$

$\qquad M = -(4L^2 w_o/\pi^2)[1 - \cos(\pi x/2L)]$

(b) $\qquad |M|_{max} = 4w_o L^2/\pi^2$

Problem 5.52

5.52 Determine (*a*) the equations of the shear and bending-moment curves for the beam and loading shown, (*b*) the maximum absolute value of the bending moment in the beam.



$$w = w_0\left(1 - \frac{x}{L}\right)$$

$$\frac{dV}{dx} = -w = -w_0 + \frac{w_0 x}{L}$$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + C_1 = \frac{dM}{dx}$$

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + C_1 x + C_2$$

$M = 0$ at $x = 0$ $\qquad C_2 = 0$

$M = 0$ at $x = L$ $\qquad 0 = -\frac{w_0 L^2}{2} + \frac{w_0 L^2}{6} + C_1 L \quad \therefore \quad C_1 = \frac{w_0 L}{3}$

$$V = -w_0 x + \frac{w_0 x^2}{2L} + \frac{w_0 L}{3}$$ ◄

$$M = -\frac{w_0 x^2}{2} + \frac{w_0 x^3}{6L} + \frac{w_0 L x}{3}$$ ◄

M is maximum where $\frac{dM}{dx} = V = 0$

$$0 = -w_0 x_m + \frac{w_0 x_m^2}{2L} + \frac{w_0 L}{3}$$

$$\frac{1}{2}x_m^2 - L x_m + \frac{1}{3}L^2 = 0 \qquad x_m = \frac{L \pm \sqrt{L^2 - (4)(\frac{1}{2})(\frac{1}{3}L^2)}}{(2)(\frac{1}{2})}$$

$$= \left(1 \pm \frac{\sqrt{3}}{3}\right)L$$

$$= \cancel{1.57735L},\ 0.42265\,L$$

$$M_{max} = \frac{-w_0(0.42265L)^2}{2} + \frac{w_0(0.42265L)^3}{6L} + \frac{w_0 L(0.42265L)}{3}$$

$$= 0.06415\ w_0 L^2$$ ◄